Order in Chaos: an Algorithmic Approach to Flocking Behaviour

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Emergence

When orderly patterns arise out of the simple interactions of chaotic entities

Essential Flocking Elements

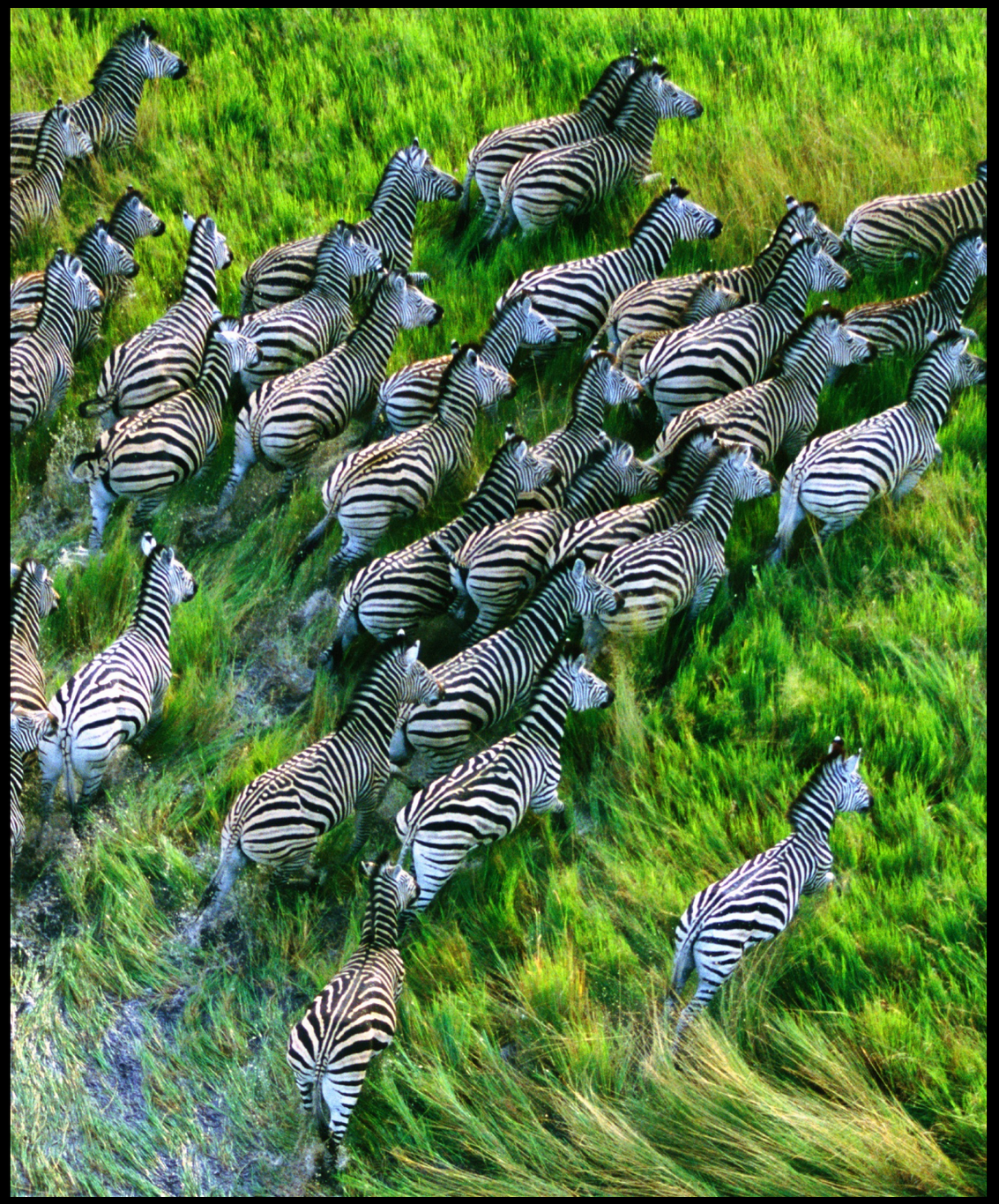
Communications (interactions)

Antagonistic behaviours (consensus and frustration)

Going beyond phase transitions

Emergence in Flocking Behaviour

- Adaptability
- Dynamic cohesion (evolving unity)
- Coherence
- Ostensive (directly demonstrate)
- Self-organisation
- “Beauty”, “Harmony” (subjective)



Boid Flocking Model

Local Rules determine Global Coherence

- Every boid has a constant speed v_0
- Individual steering determined as an <u>average</u> of flock mate directions

Two-Step Dynamics

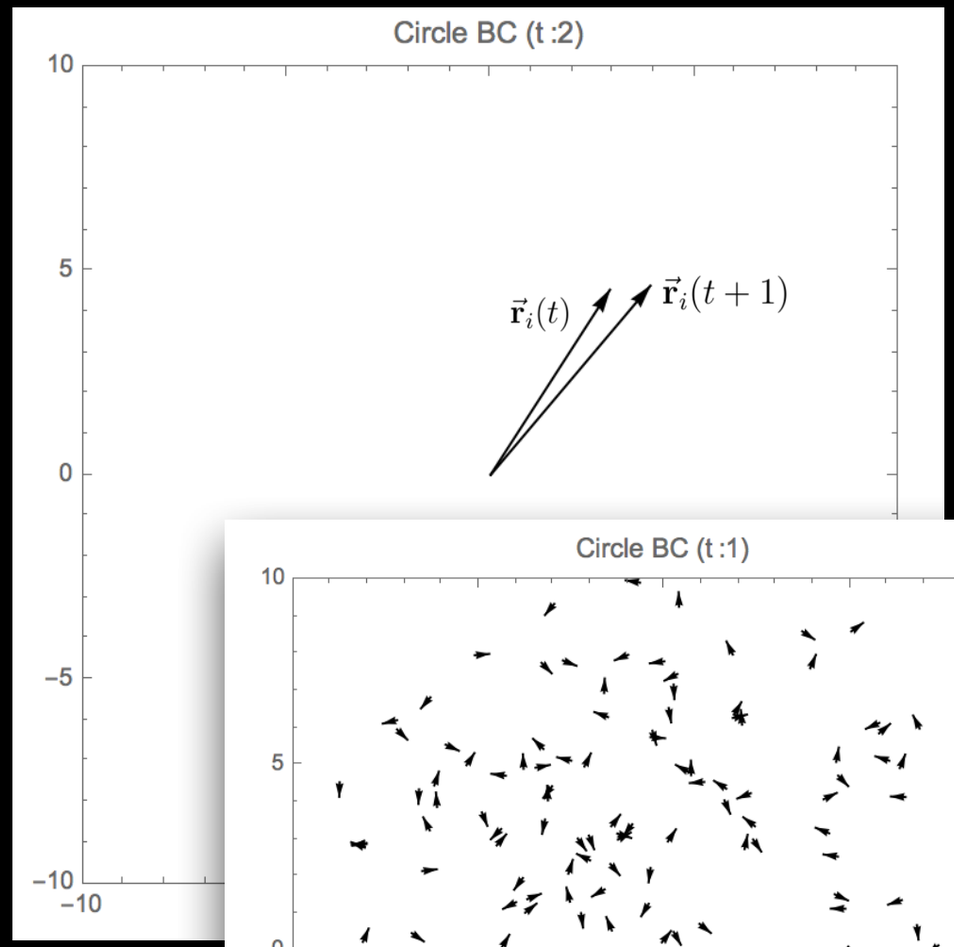
- Choose a small fixed number of flock mates for each boid

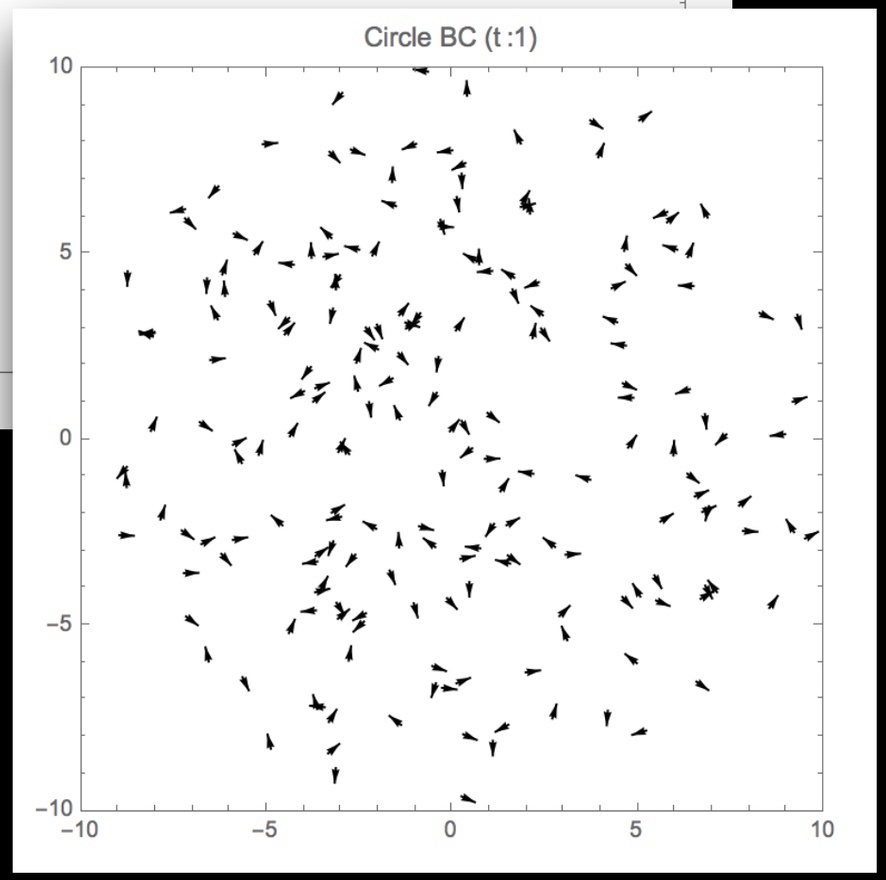
1. Calculate the boid's steering averaging with its flock mates'
2. Update all positions with a finite step, imposing boundary conditions

Creating the Environment (2D)

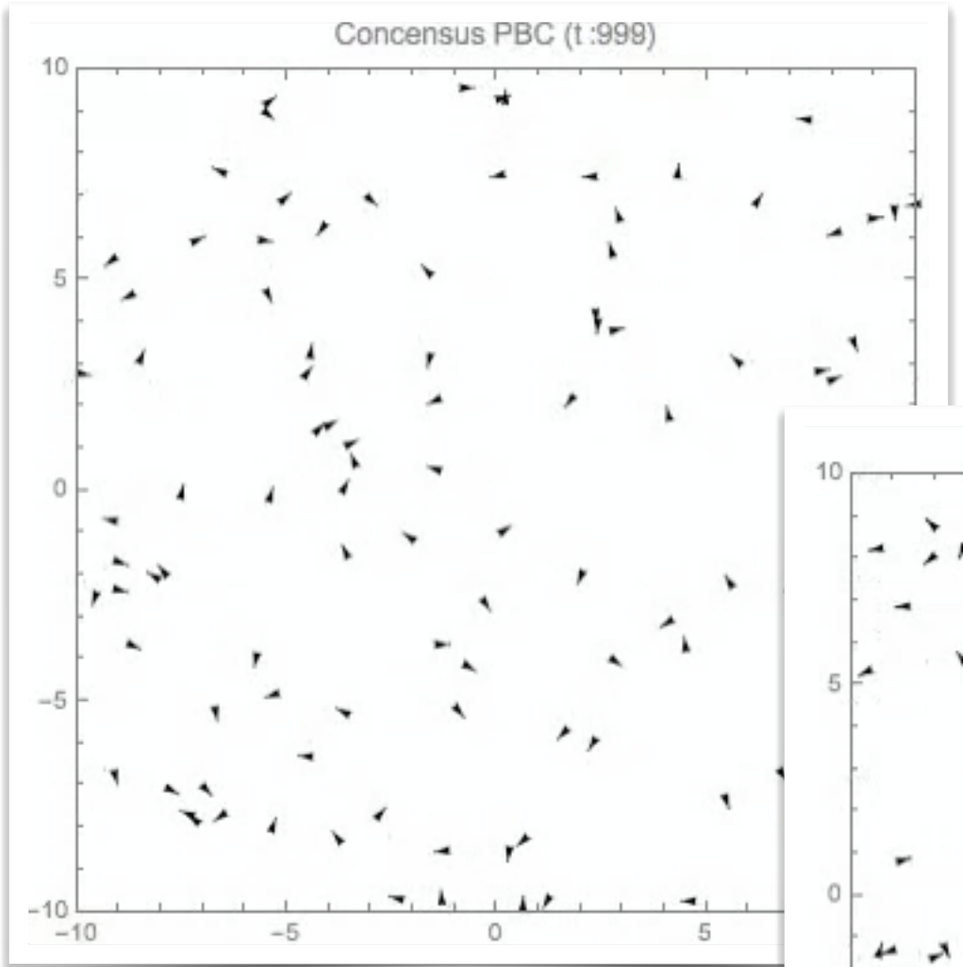
$$\vec{\mathbf{v}}_i(t+1) = v_0 f_i(\hat{v}(t))$$
$$\vec{\mathbf{r}}_i(t+1) = \vec{\mathbf{r}}_i(t) + \vec{\mathbf{v}}_i(t+1)\Delta t$$

- *i* = labels the boid
- *t* = labels the time step
- f_i = average the velocity with flock mates at each time step
- Random starting positions
- Random starting directions

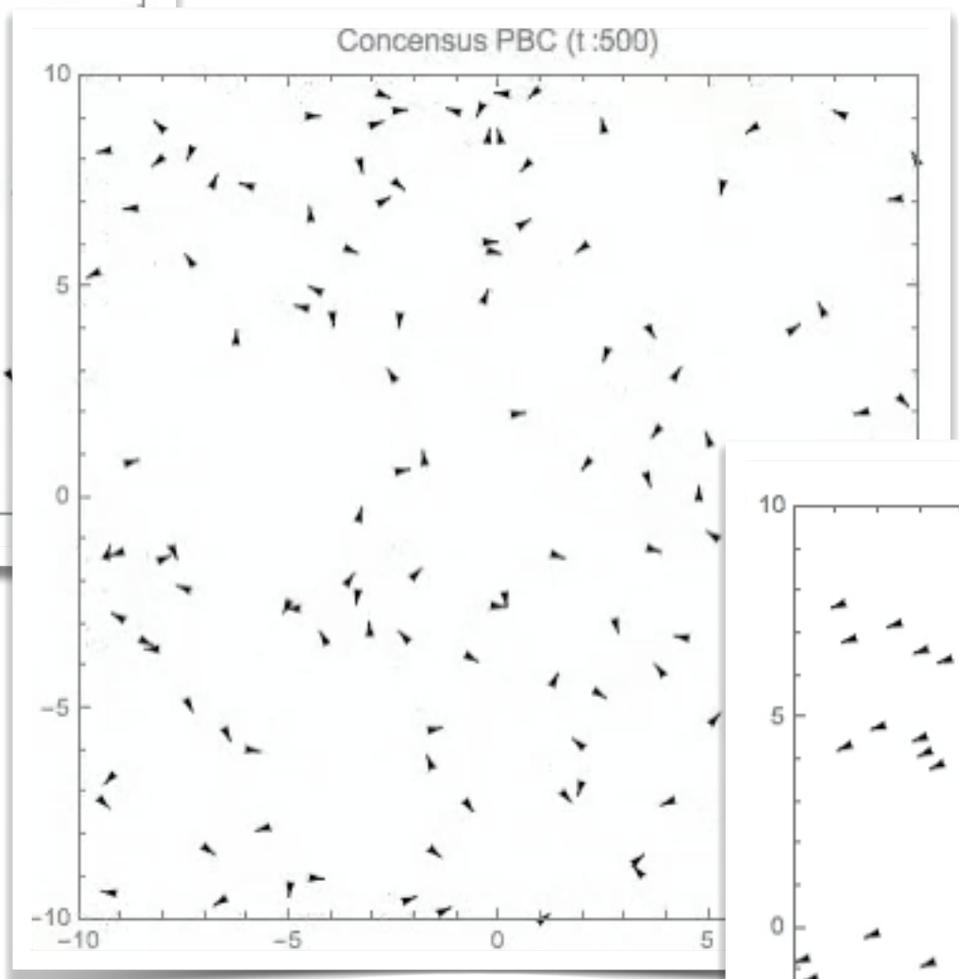




3 Unique Phases

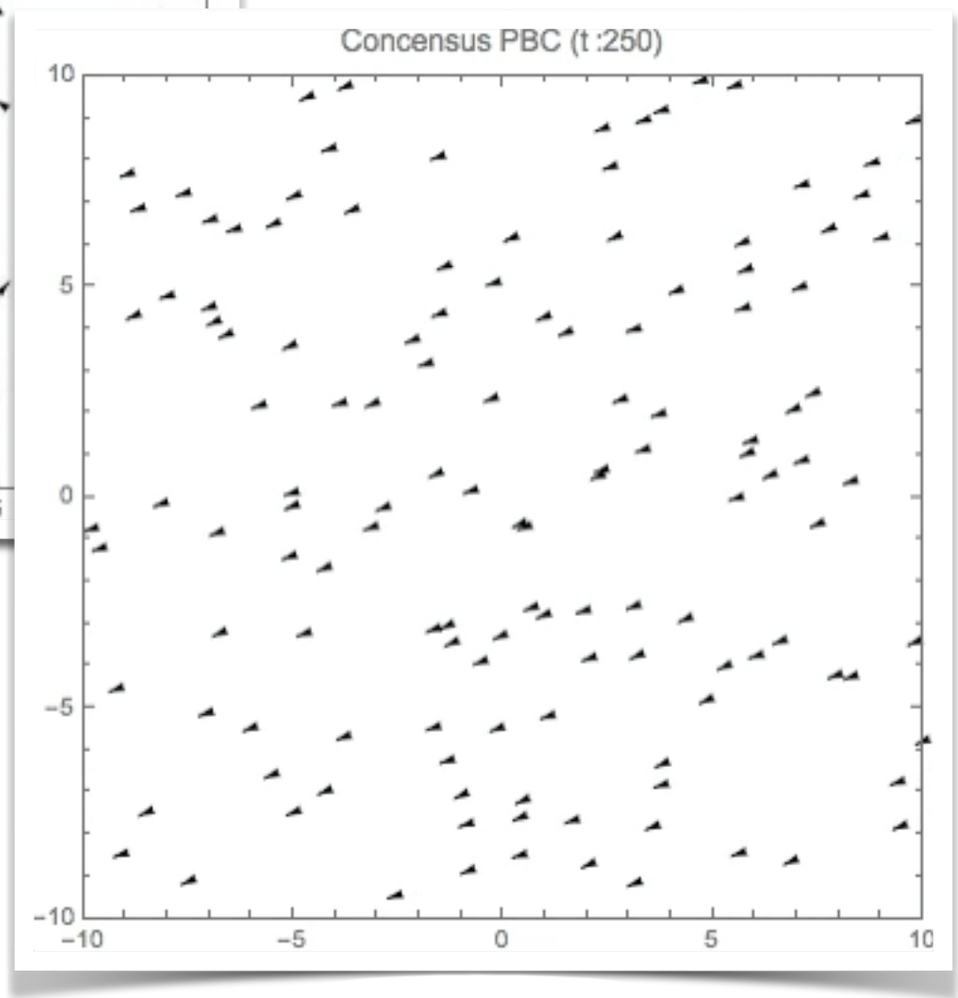


Clockwise (-)

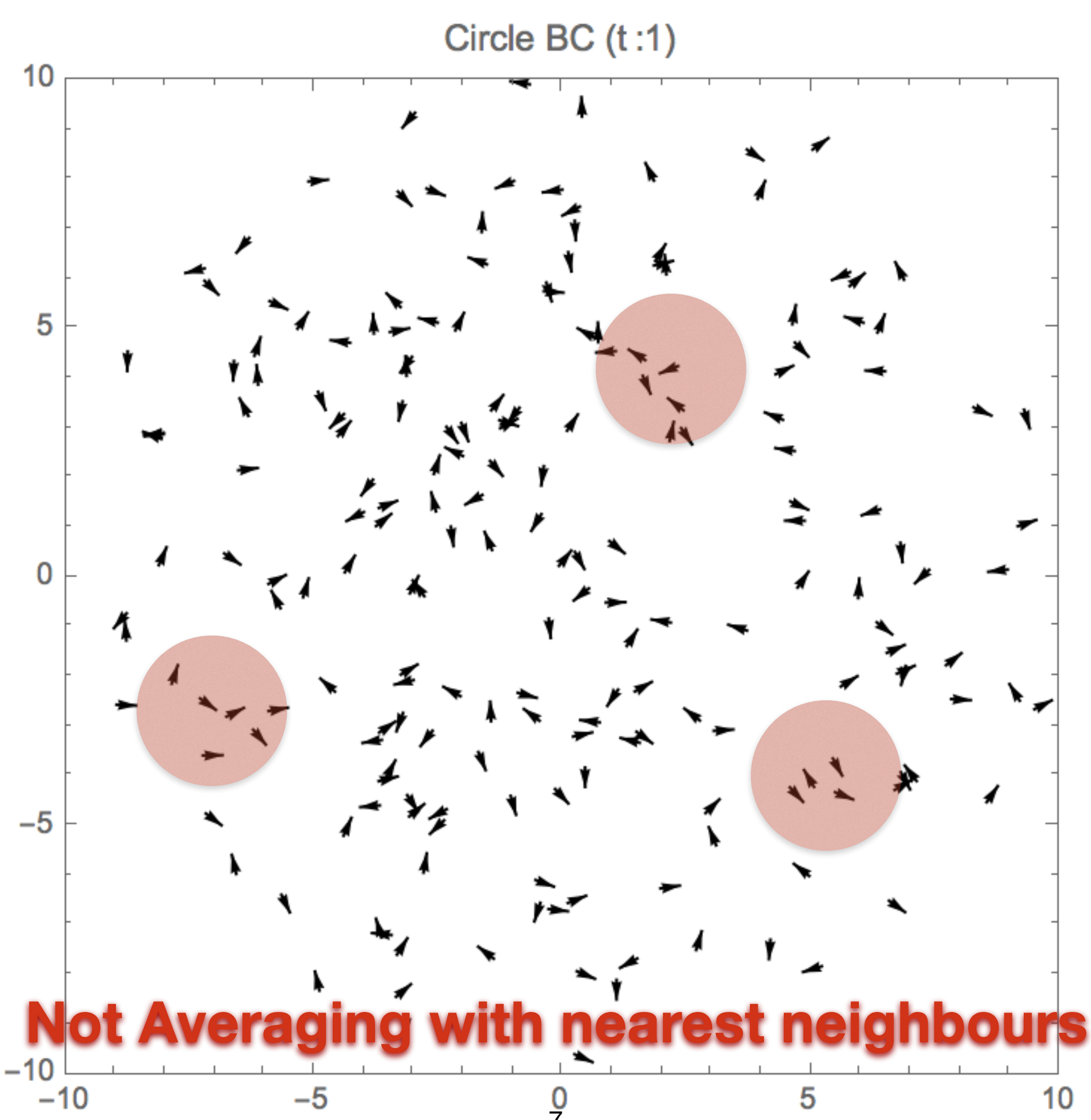


Counter-Clockwise (+)

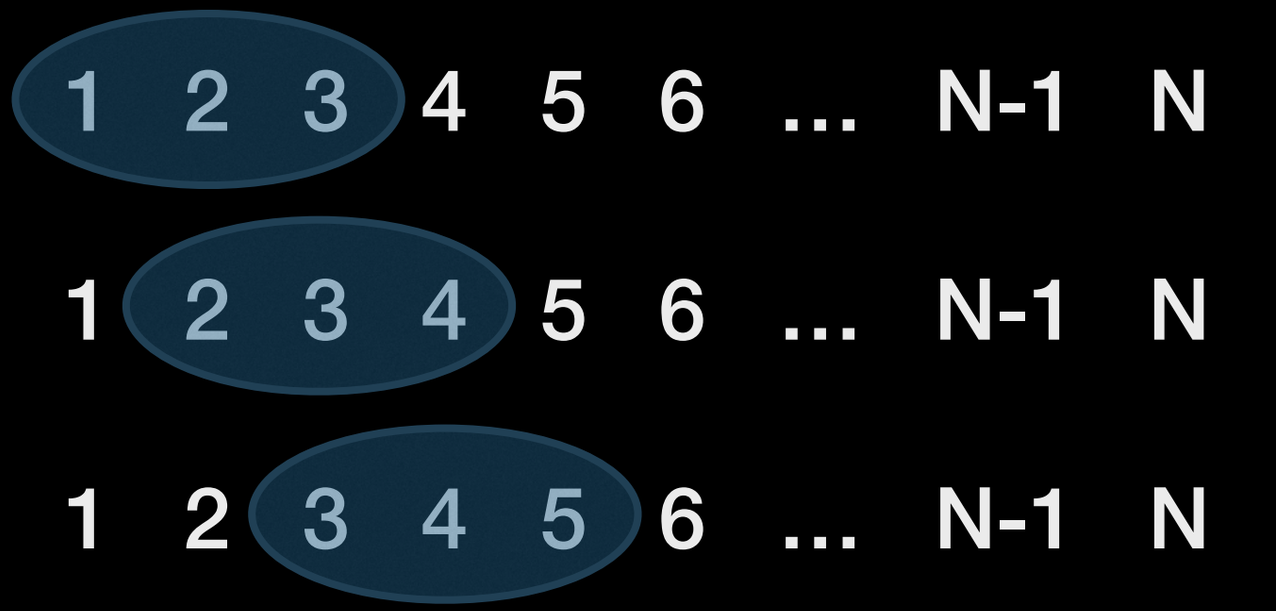
Aligned (0)



- **Technical Point: periodic boundary conditions**


Circle BC (t :1)
10
5
0
−5
−10
−10
−5
0
5
10
Not Averaging with nearest neighbours

Steering Average Partners



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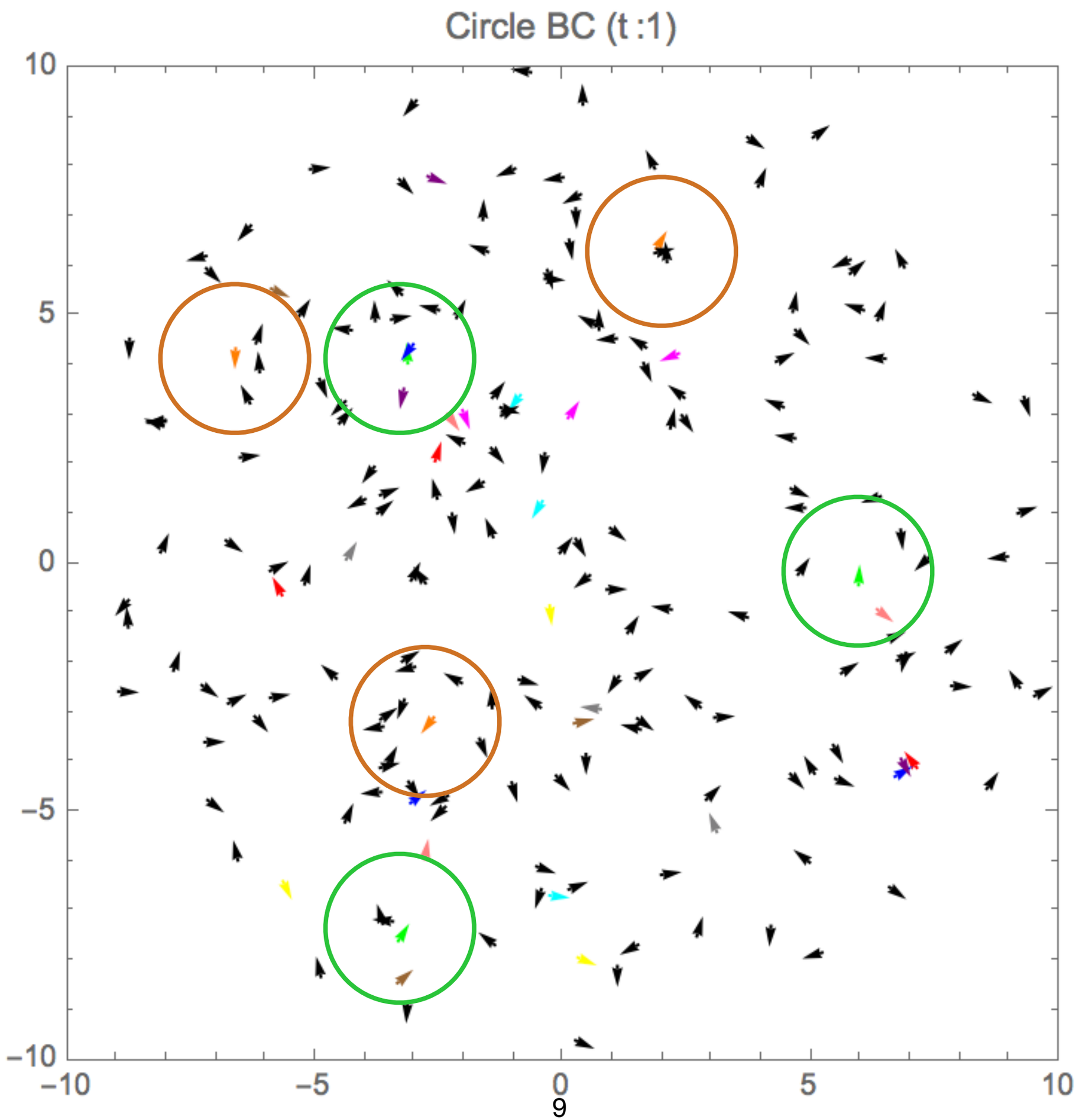
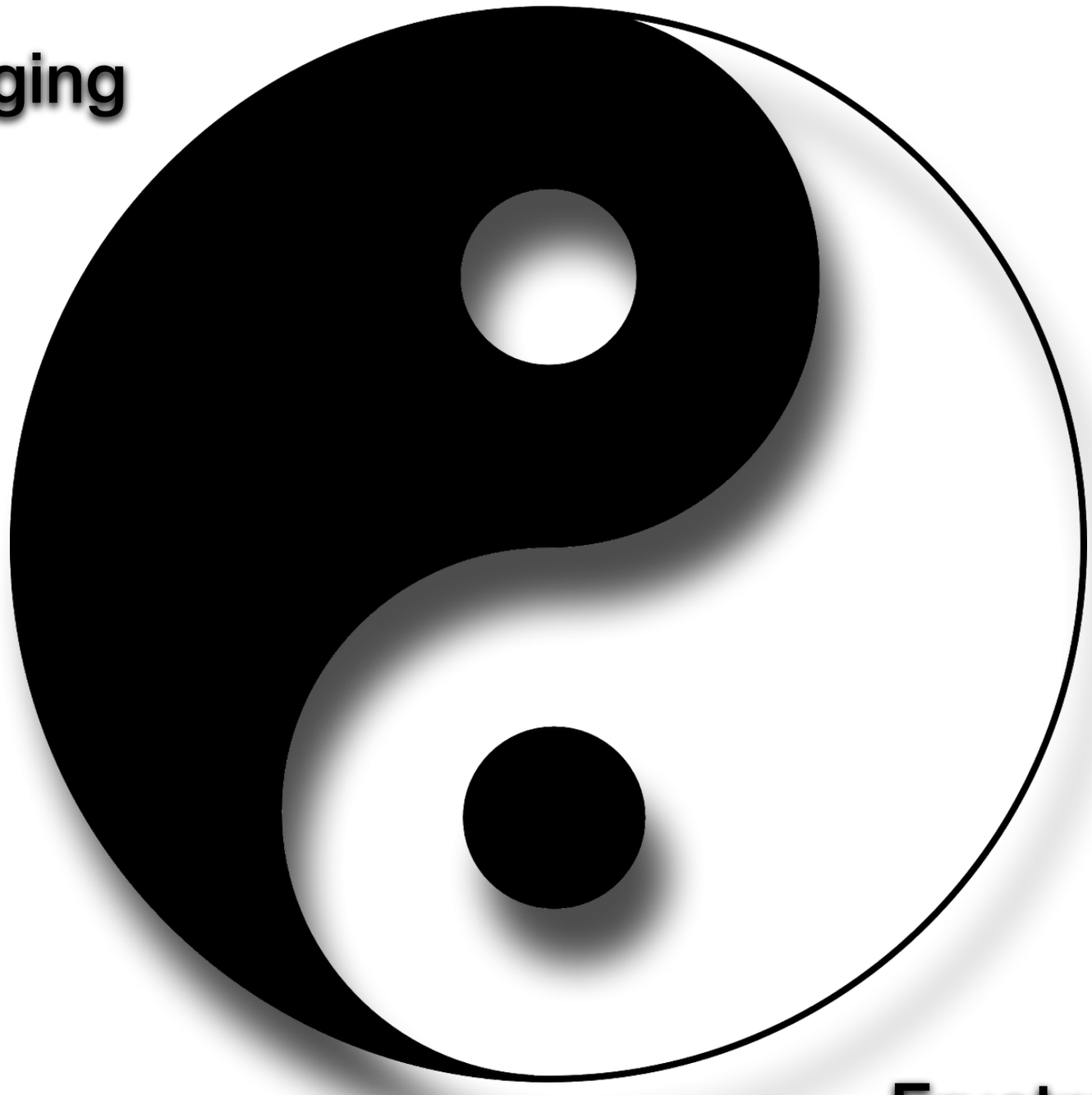
Democratic but not reciprocal, 1 ► 2 does not imply 2 ► 1

Flock Mates


Circle BC (t :1)
10
5
0
−5
−10
−10
−5
0
5
10

Consensus
Velocity Averaging



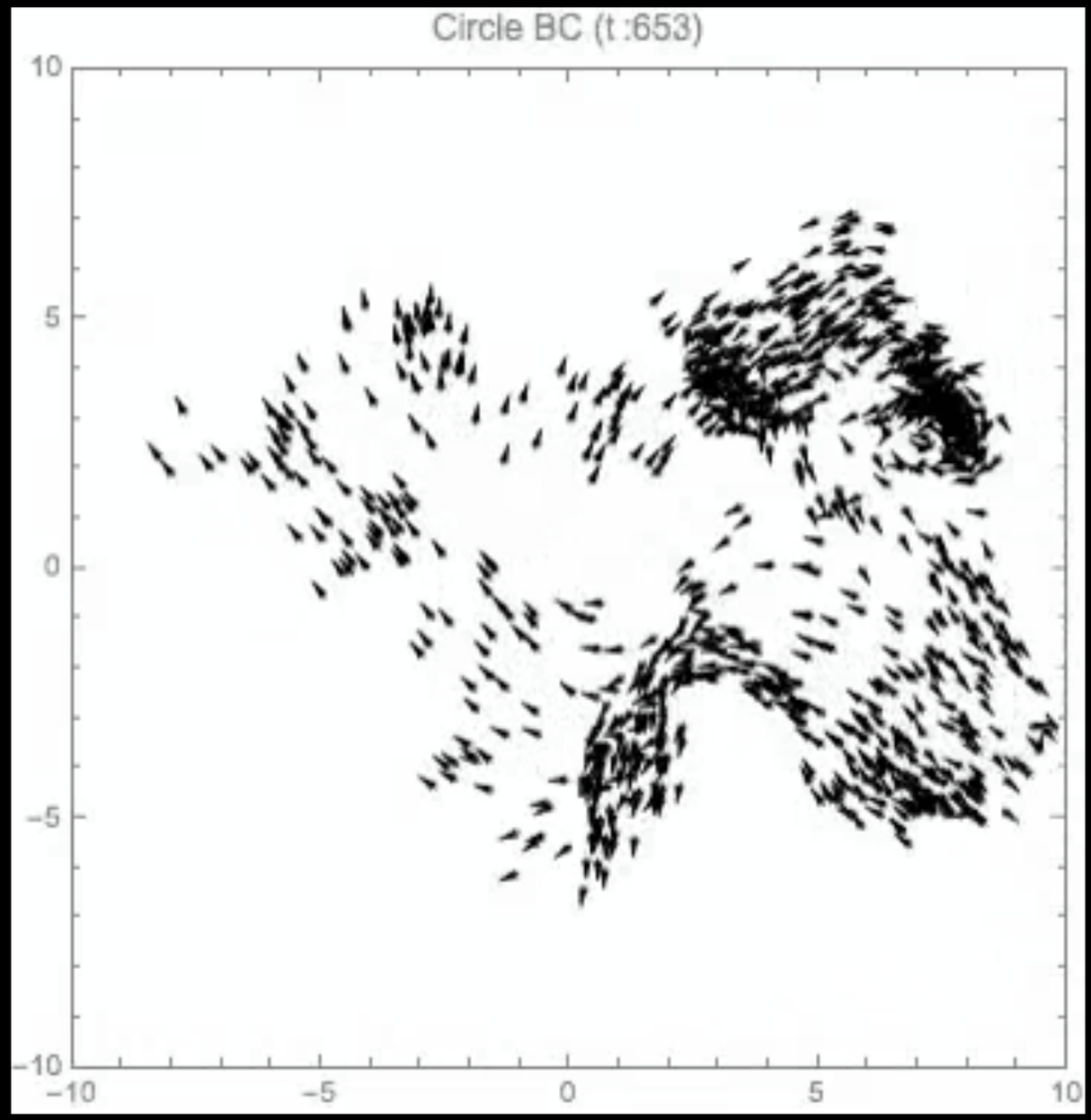
Frustration
Boundary Conditions

Add Frustration

Boundary conditions

- The antagonistic forces of consensus and frustration take interactions beyond simple phase transitions

Emergence



Order Parameters

a) Alignment Order Parameter (OP1)

$$\langle v(t) \rangle = \frac{1}{N v_0} \left| \sum_{i=1}^{N} \vec{\mathbf{v}}_i(t) \right|$$

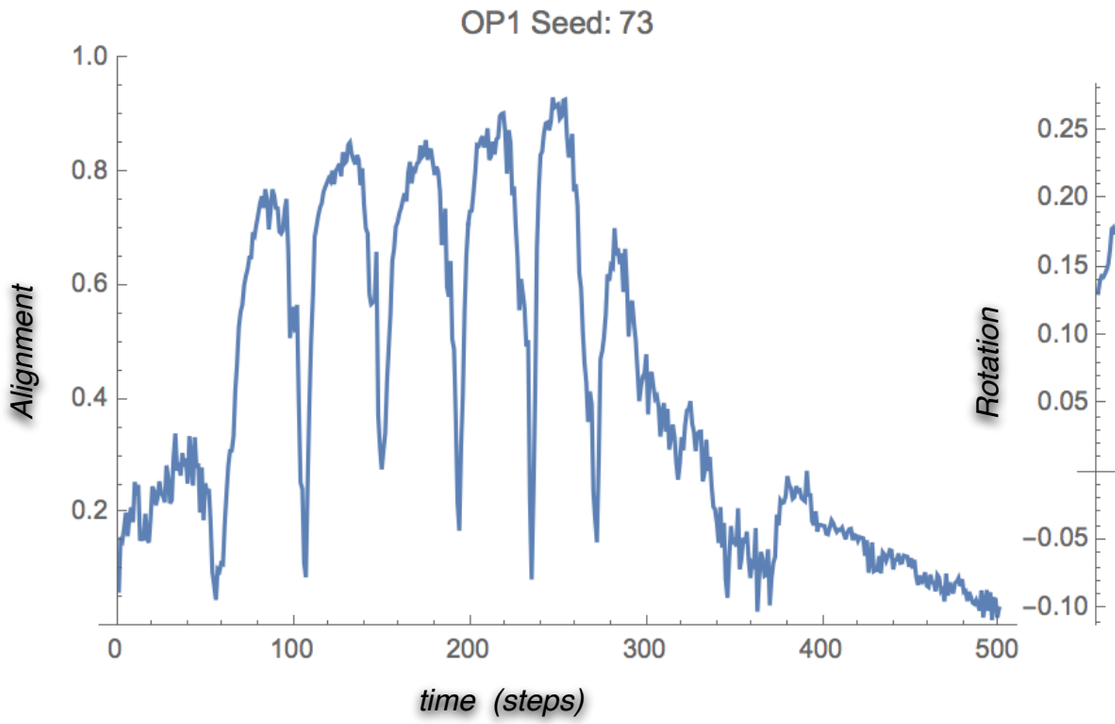
1 if all boids are aligned
0 if random or rotating

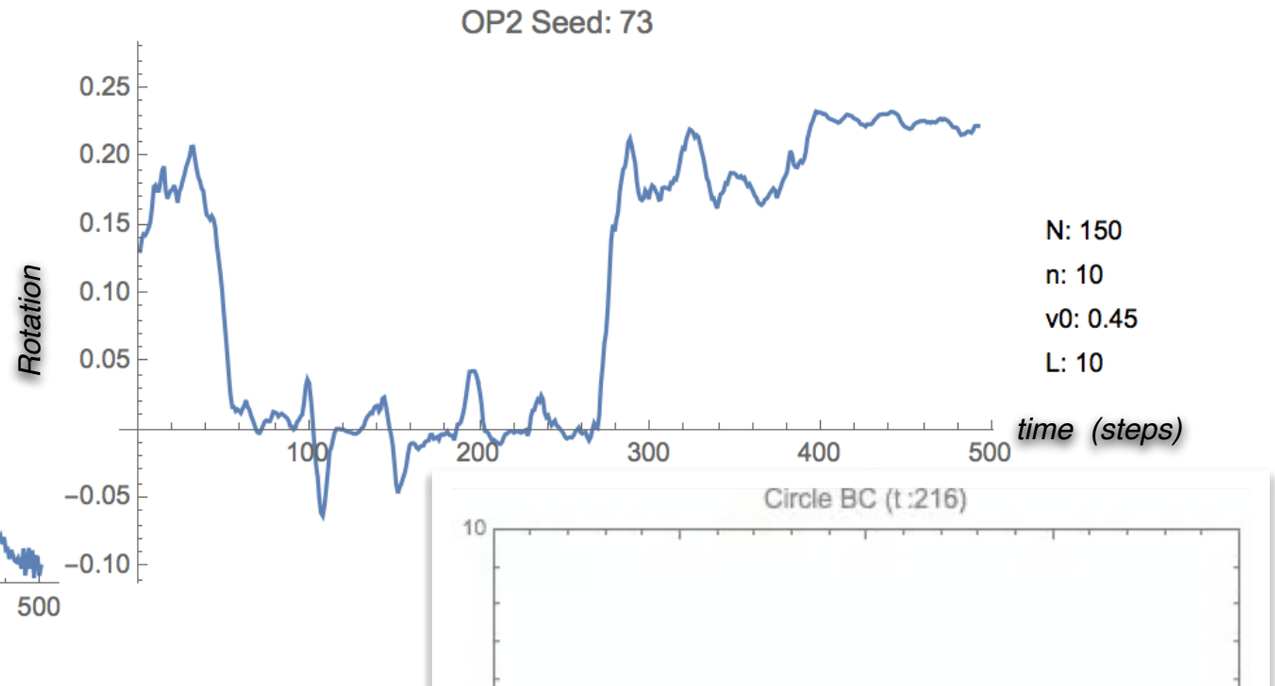
b) Rotational Order Parameter (OP2)

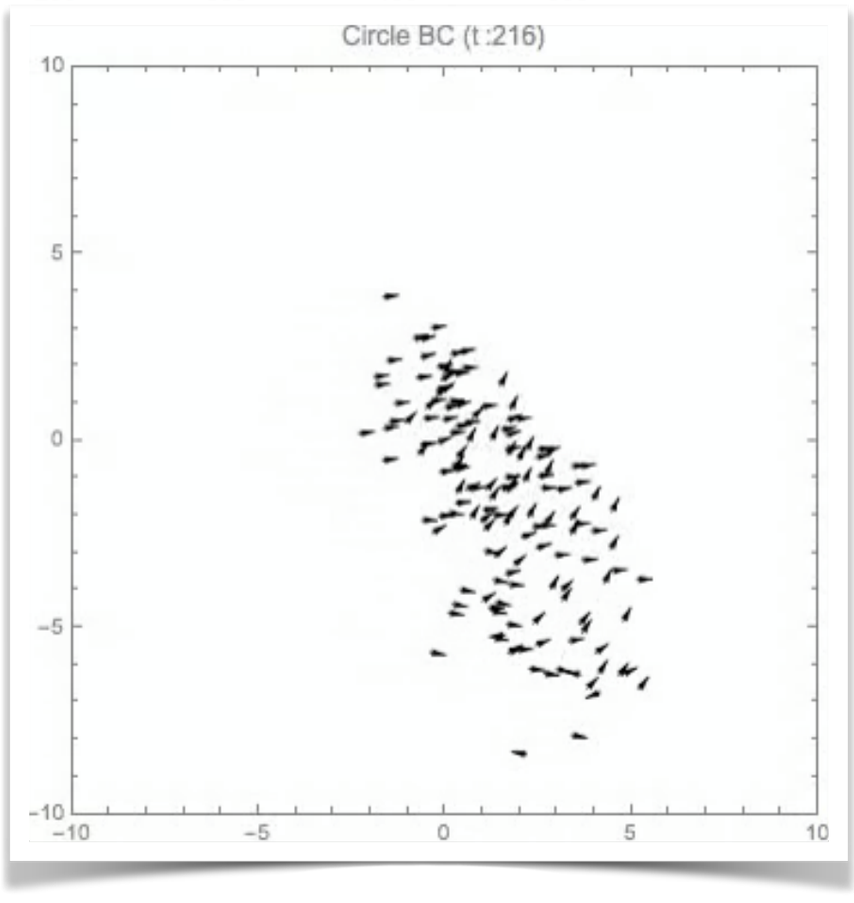
$$L(t) = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{z} \sum_{t=1}^{z} \frac{v_i(t) \wedge v_i(t+1)}{v_0^2}$$

N is the total number of boids

Order Parameters and Phase Transitions

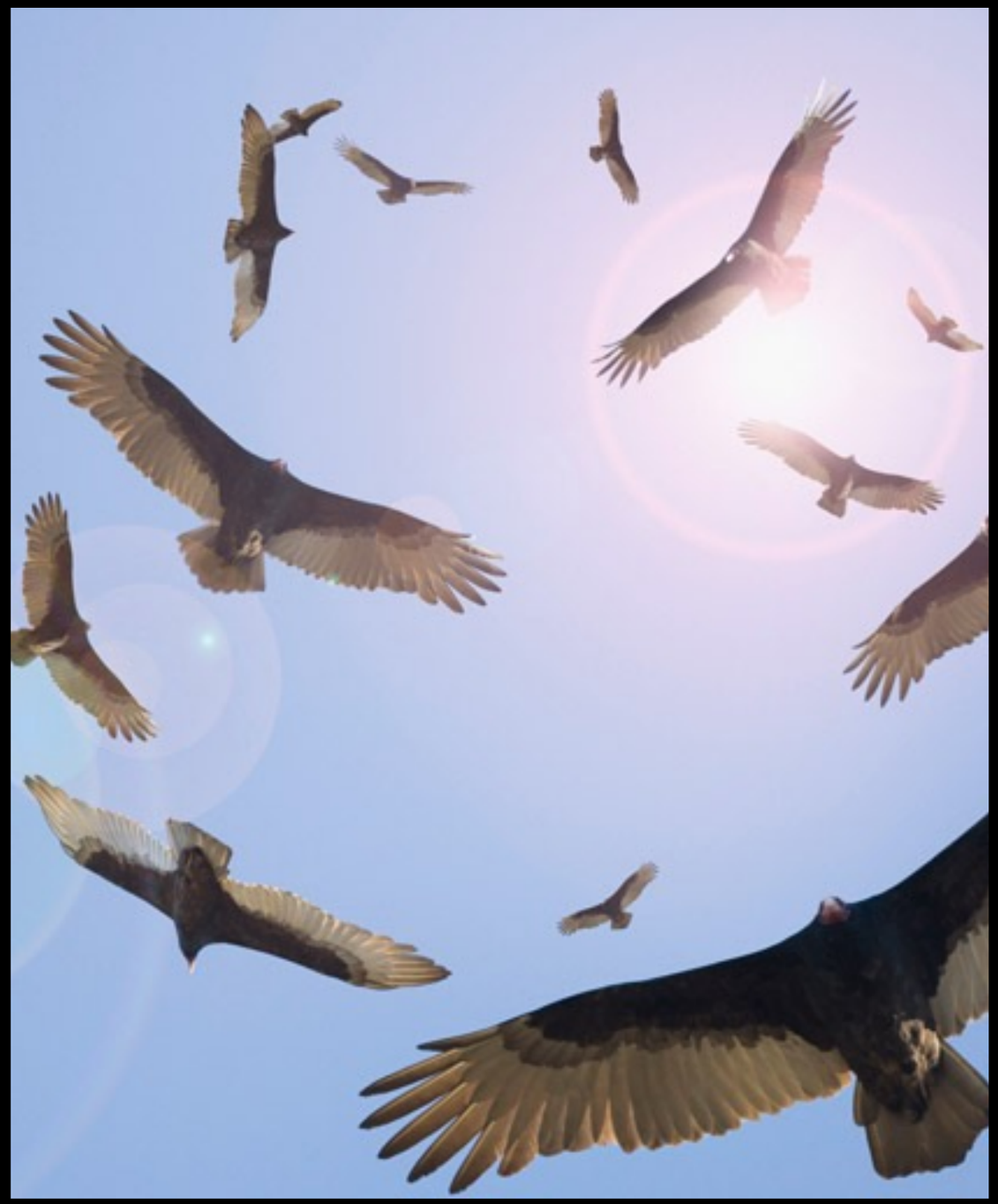






- **When the alignment is low, the system is either rotating or random**

Summary



- Consensus comes from averaging velocities while frustration comes from boundary conditions
- Each time step averages the velocity with each boid's flock mates. Boundary conditions are taken into consideration
- The antagonism between the consensus and frustration creates the emergence