

**Abstract:** The emergent behavior of certain collective systems such as starling murmurations reveals coherent behavior arising from the simple, individual interactions of its entities. Using a two-dimensional algorithmic model, we can show that self-driven particles (boids) group together and display emergent flocking characteristics. The model is based on the ideas of consensus and frustration as well as the dynamic interplay between global and local phase transitions. The frustration is a perturbation that drives the boids beyond the simple phase transitions and towards chaotic behavior while the consensus is a topological averaging, that balances the frustration. The results are interpreted in terms of global and local order parameters, and correlation functions.

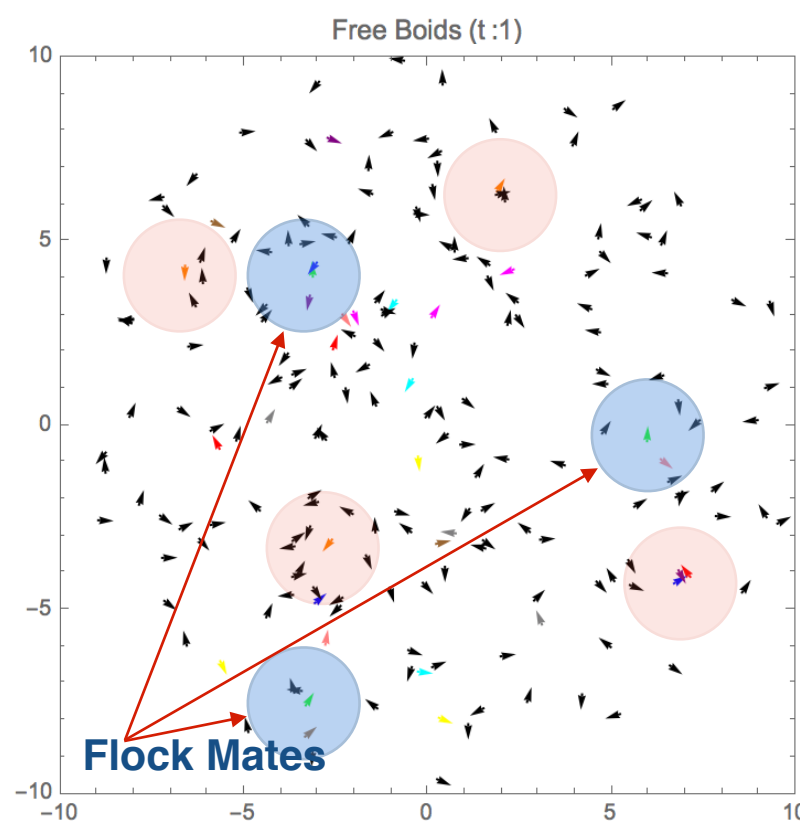
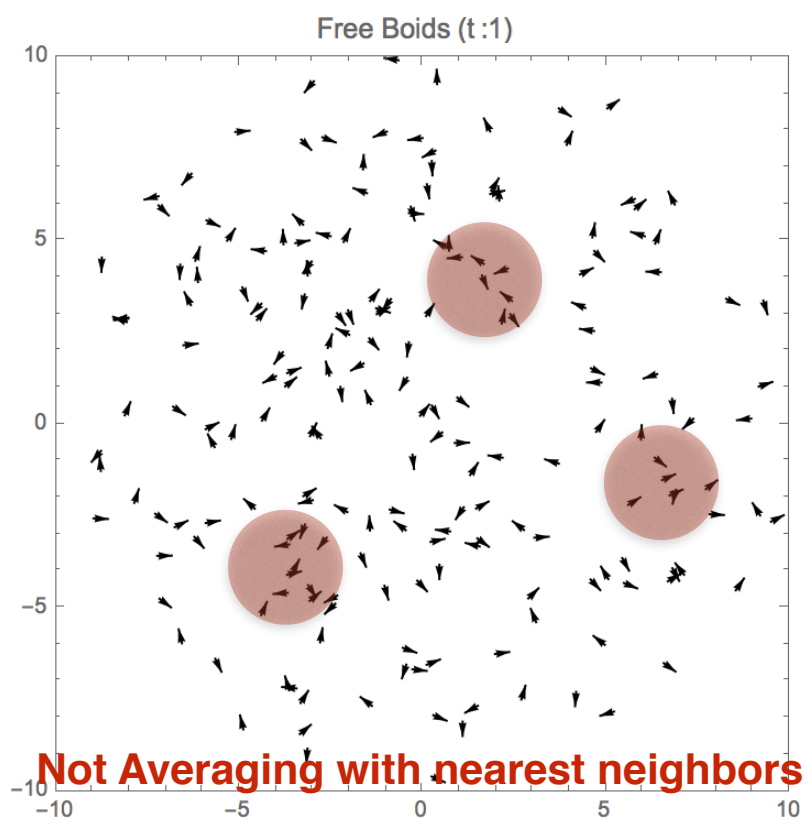
## Emergence: Orderly patterns from chaotic elements

Essential Flocking Elements:

- Communications (interactions)
- Antagonistic Behaviors (consensus and frustration)
- Going beyond phase transitions

## Boid Flocking 2D Model

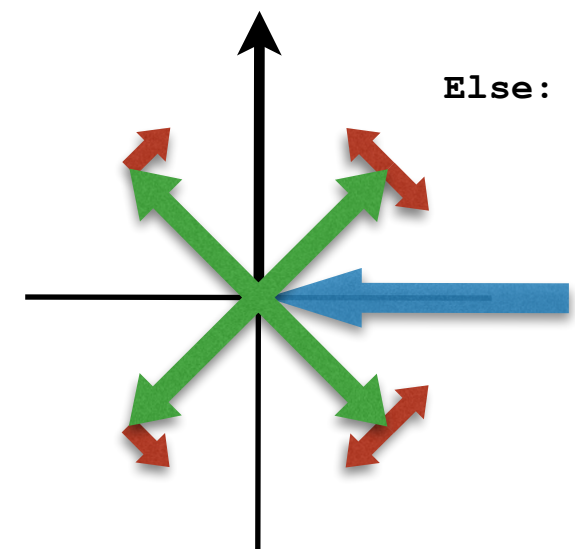
1. Average velocities while imposing frustration
  - Steering is a perturbed average of flock mates
  - Constant speed  $v_0$
2. Update position



## Implementing Frustration

$$\text{If: } \mathbf{v}_i(t) \cdot \mathbf{r}_i(t) > 0 : \theta = \left( \frac{|\mathbf{r}_i(t)|}{L} \right)^p * \text{Sign}(\mathbf{v}_i(t) \cdot \mathbf{T}_i(t)) * |\eta|$$

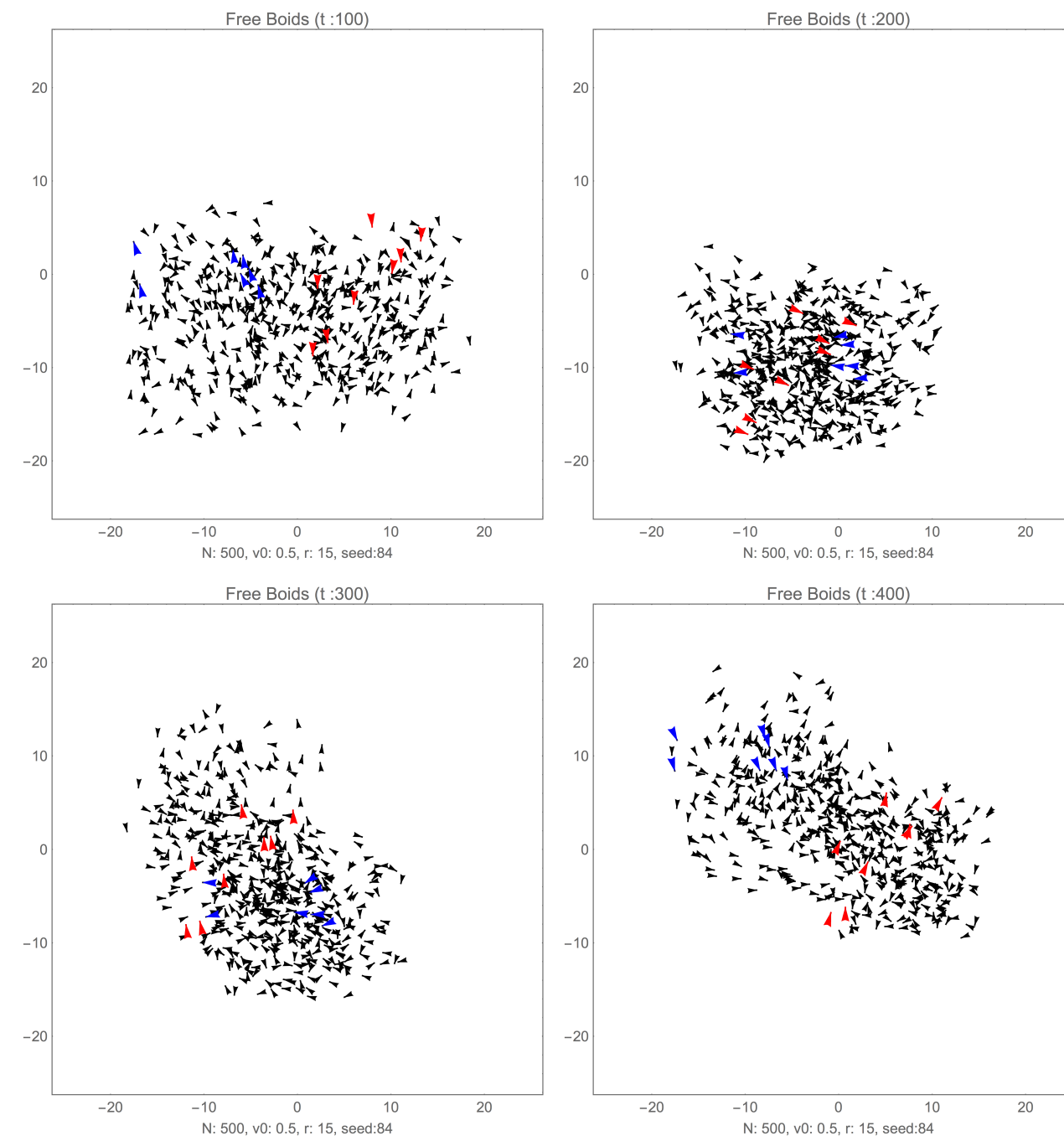
$$\text{Else: } \theta = \left( \frac{|\mathbf{r}_i(t)|}{L} \right)^p * \eta$$



- $\mathbf{r}_i(t)$  : position
- $\mathbf{v}_i(t)$  : velocity
- $\theta$  : frustration
- $\mathbf{T}_i(t)$  : tangential

$\eta$  : normally distributed random number

## Time Evolution



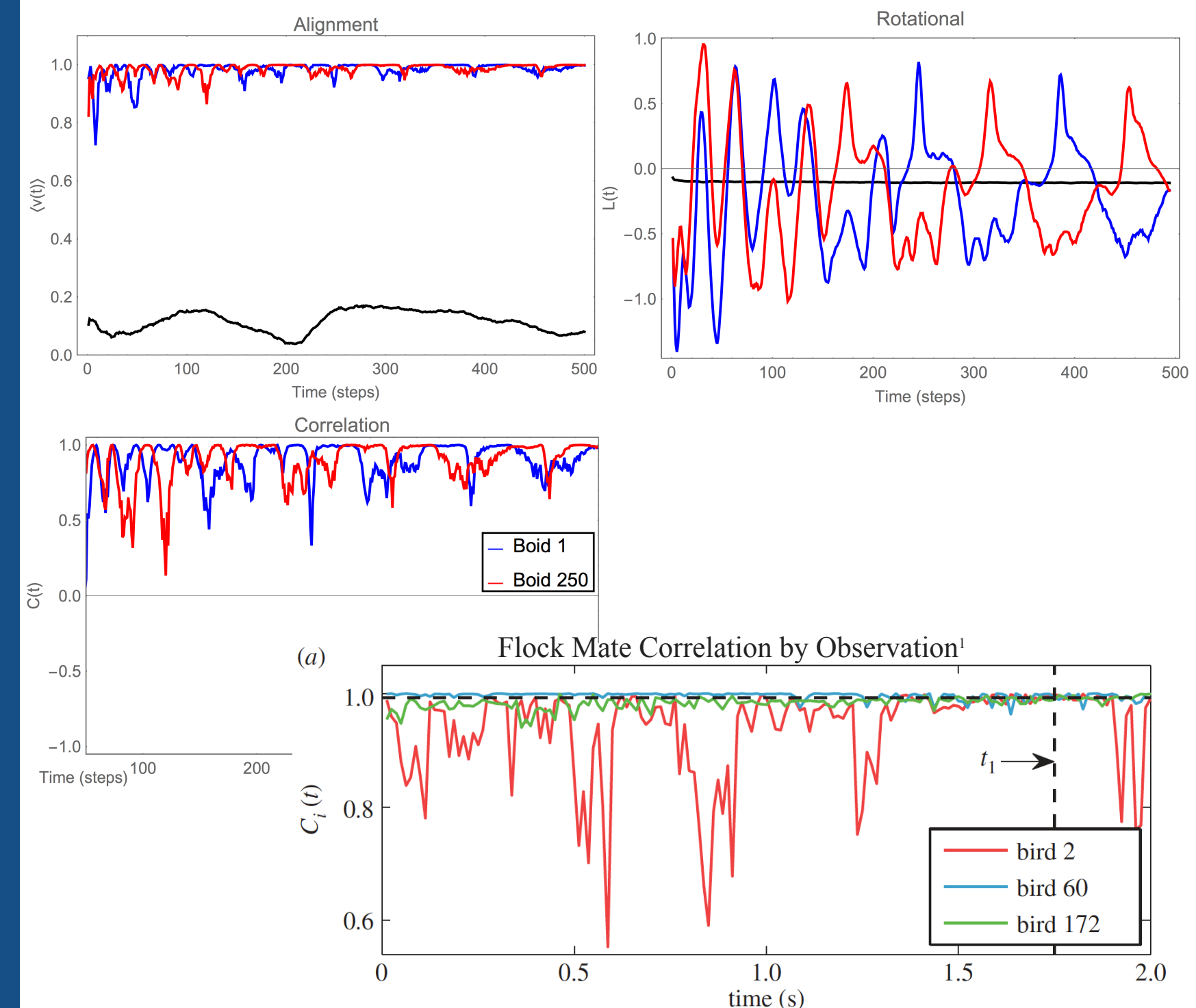
## Order Parameters

$$\langle v(t) \rangle = \frac{1}{N v_0} \left| \sum_{i=1}^N \vec{v}_i(t) \right| \quad \begin{array}{l} 1 \text{ if aligned} \\ 0 \text{ if unaligned or random} \end{array}$$

$$L(t) = \frac{1}{N} \sum_{i=1}^N \frac{1}{z} \sum_{t=1}^z \frac{\mathbf{v}_i(t) \wedge \mathbf{v}_i(t+1)}{v_0^2} \quad \begin{array}{l} (-) : \text{Clockwise} \\ 0 : \text{random or no rotation} \\ (+) : \text{Counter-Clockwise} \end{array}$$

$$C_i(t) = \frac{\mathbf{v}_i(t)}{v_i(t)} \cdot \frac{\mathbf{V}_n(t)}{V_n(t)} \quad \begin{array}{l} 1 : \text{strongly correlated} \\ -1 : \text{anti-correlated} \end{array}$$

## Results



## Conclusion

Velocity averaging causes phases, frustration provides a propensity to chaos, and a delicate balance between the two results in emergence that resembles flocking behavior observations.

1. Attanasi, Alessandro, Andrea Cavagna, Lorenzo Del Castello, Irene Giardina, Asja Jelic, Stefania Melillo, Leonardo Parisi, Oliver Pohl, Edward Shen, and Massimiliano Viale. "Emergence of collective changes in travel direction of starling flocks from individual birds' fluctuations." *Journal of The Royal Society Interface* 12, no. 108 (2015): 20150319.