

Beyond Phase Transitions: an Algorithmic Approach to Flocking Behavior



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Emergence

Essential Flocking Elements:

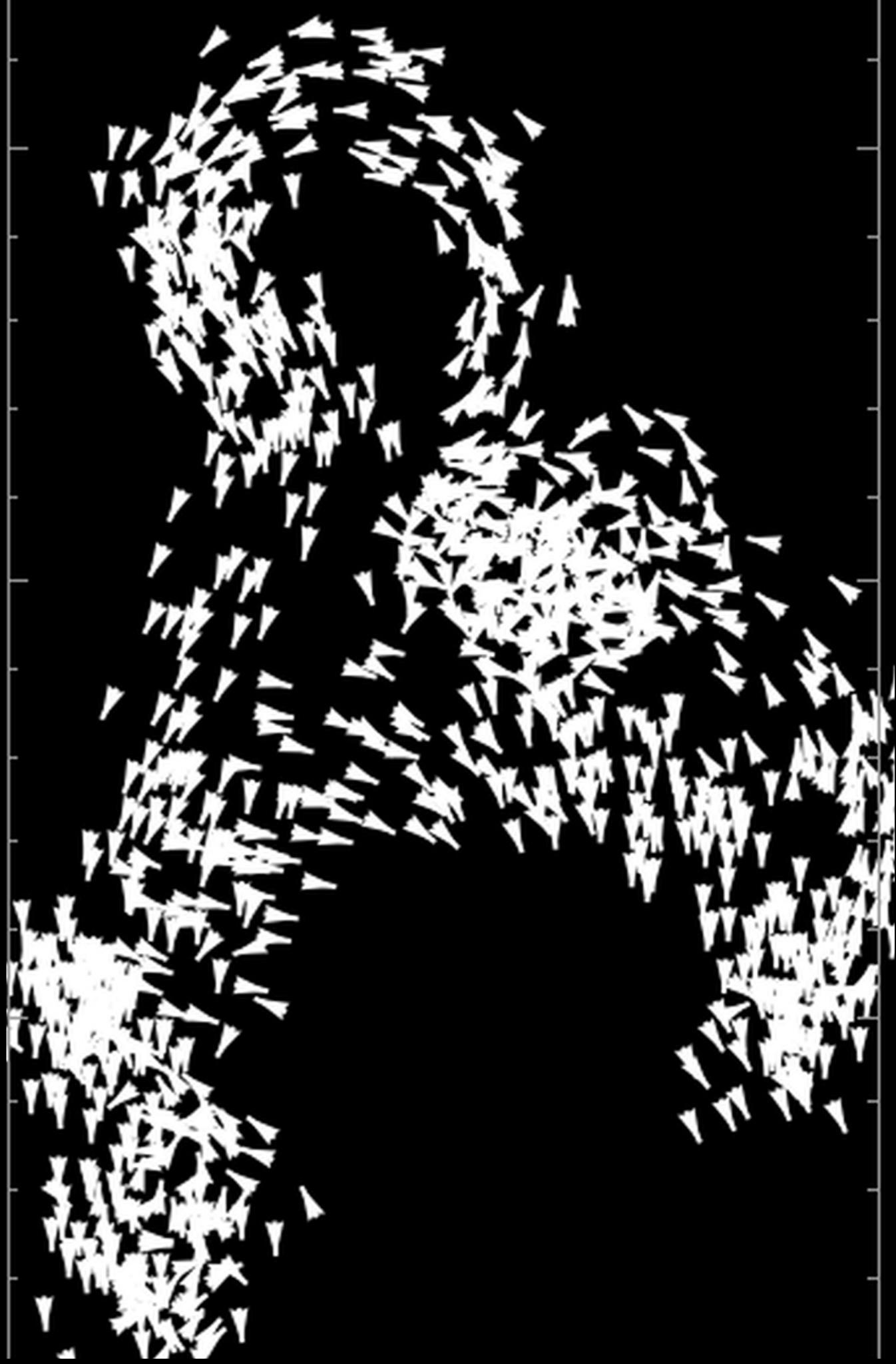
- Communications (interactions)
- Antagonistic behaviors (consensus and frustration)
- Going beyond phase transitions



Boid Flocking 2D Model

Two-Step Dynamics

- Constant speed v_0
- 1. Average velocities while imposing frustration
 - Steering is a perturbed, average of flock mates
- 2. Update positions

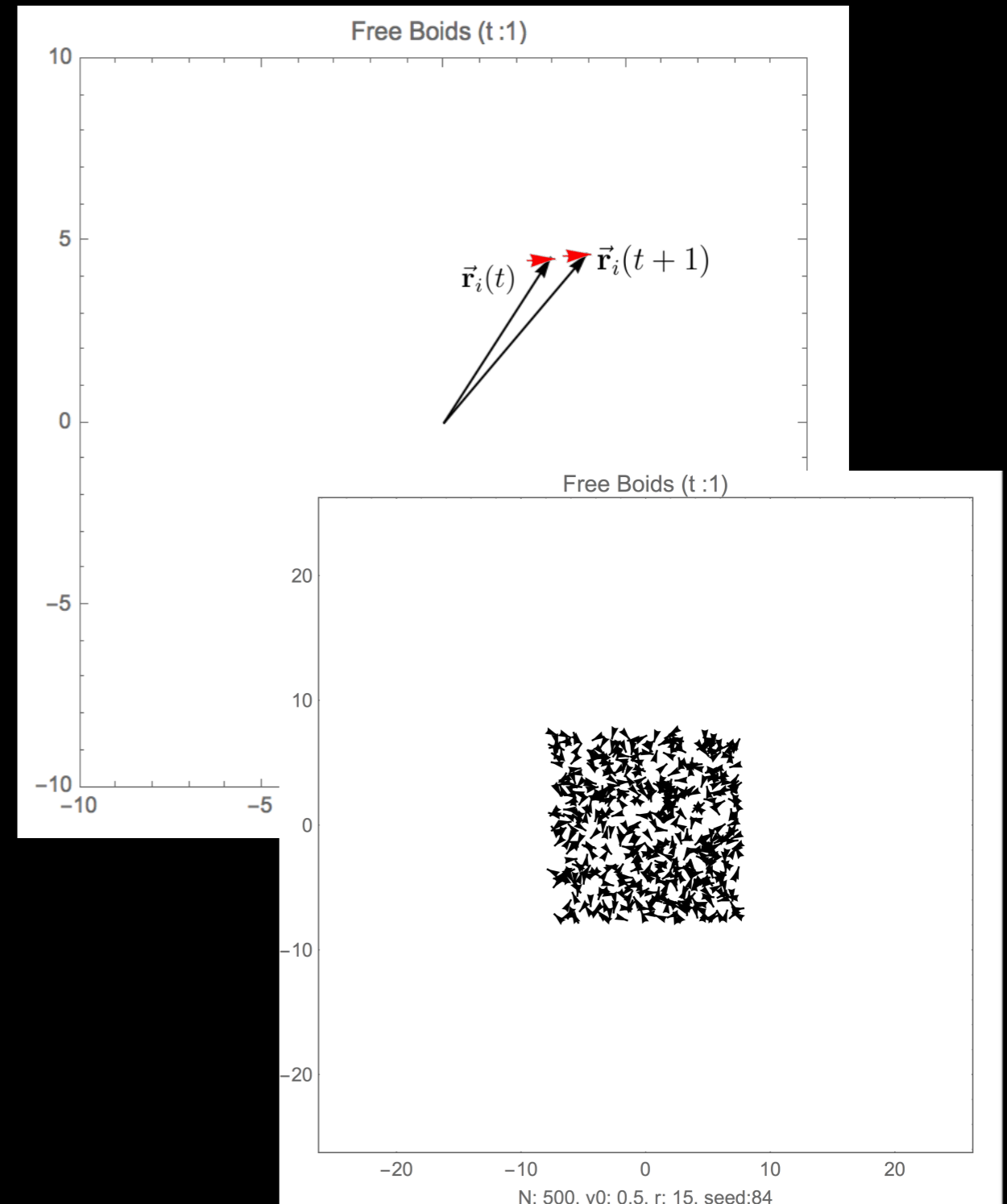


Creating the Environment (2D)

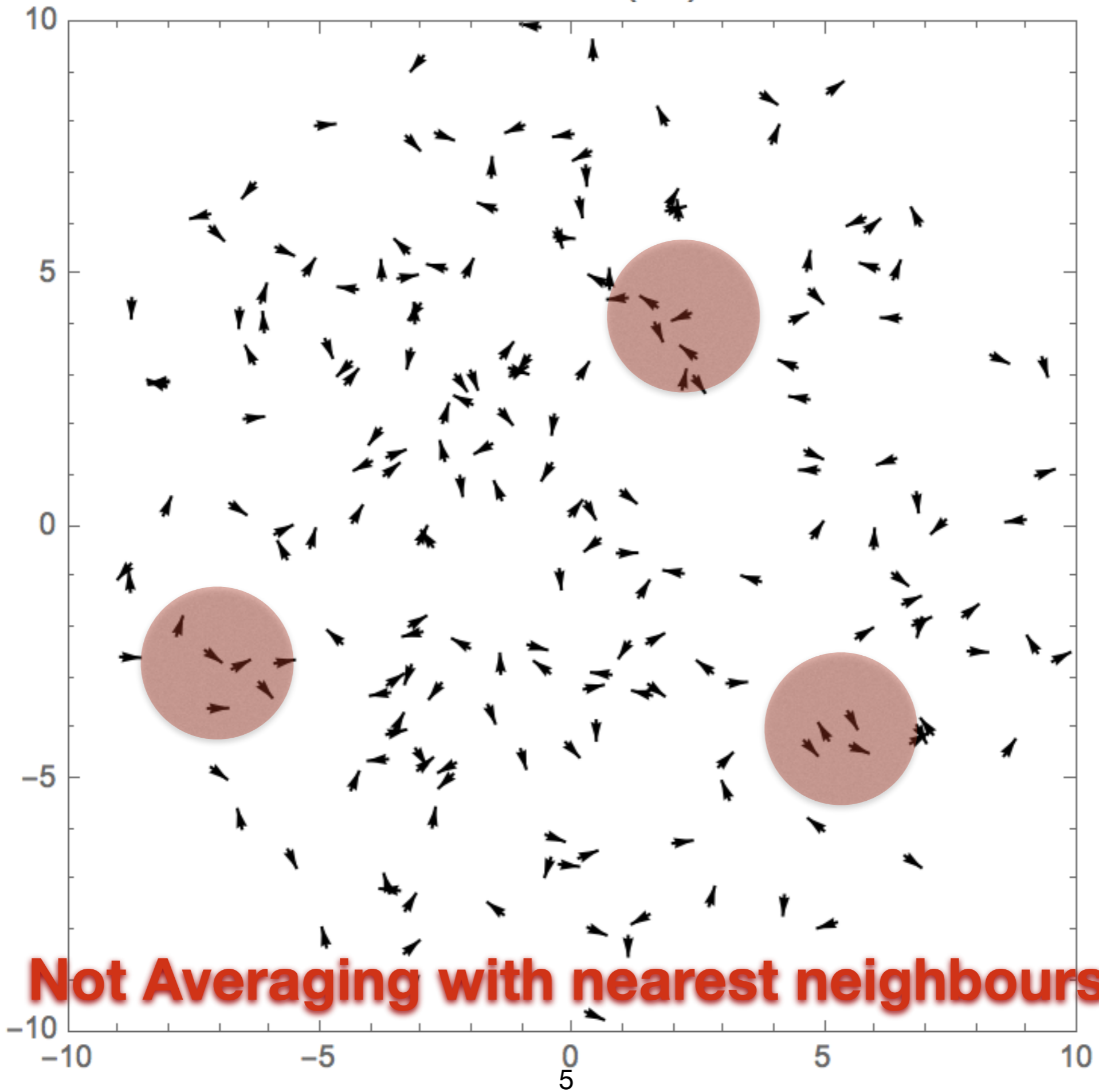
$$\vec{v}_i(t+1) = v_0 f_i(\hat{v}(t))$$

$$\vec{r}_i(t+1) = \vec{r}_i(t) + \vec{v}_i(t+1)\Delta t$$

- Random starting positions
- Random starting directions

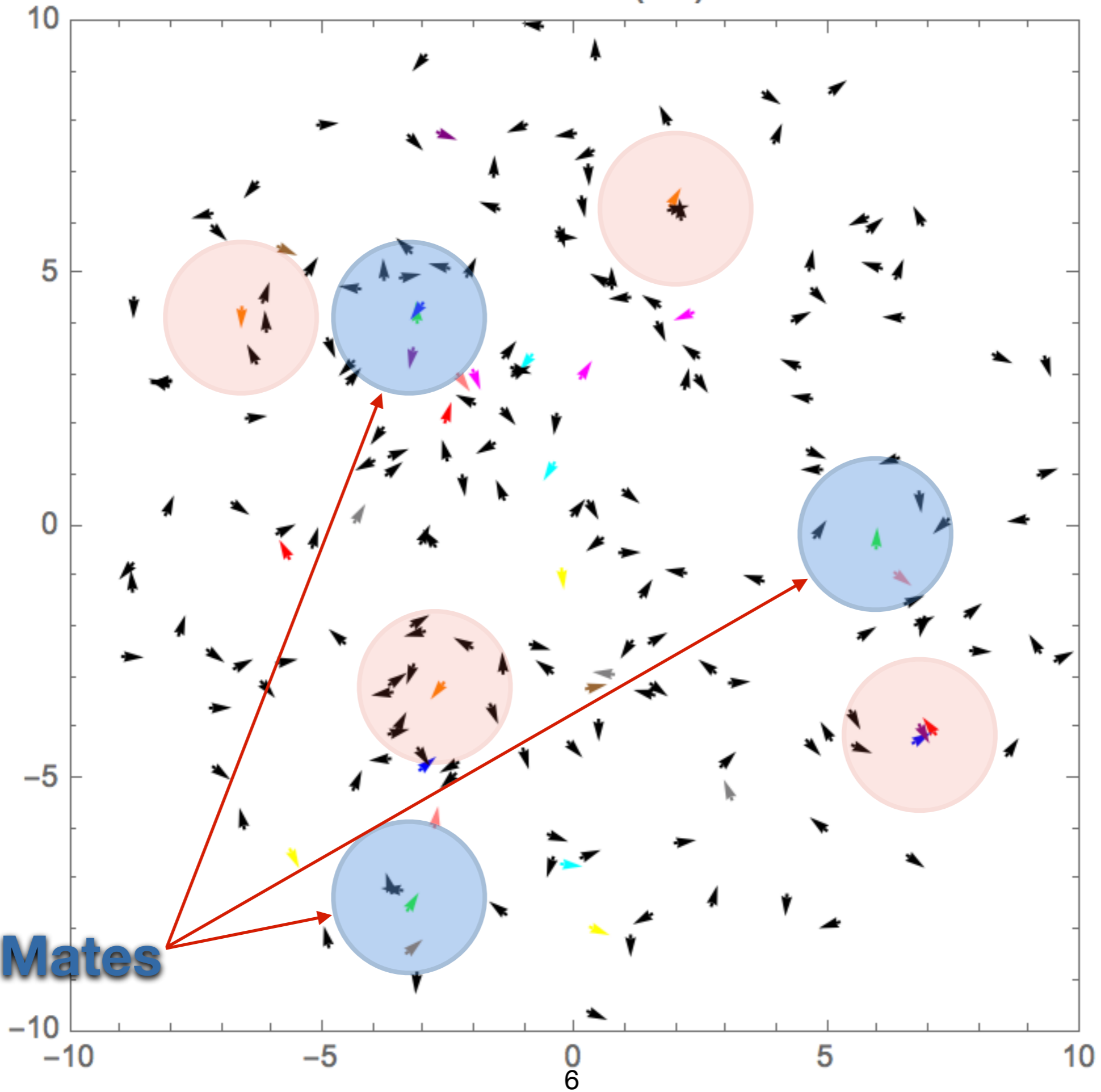


Free Boids (t :1)



Not Averaging with nearest neighbours

Free Boids (t:1)



Flock Mates

Steering Average Partners

1 2 3 4 5 6 7 8 9 10 11 12 ... N-1 N

1 2 3 4 5 6 7 8 9 10 11 12 ... N-1 N

1 2 3 4 5 6 7 8 9 10 11 12 ... N-1 N

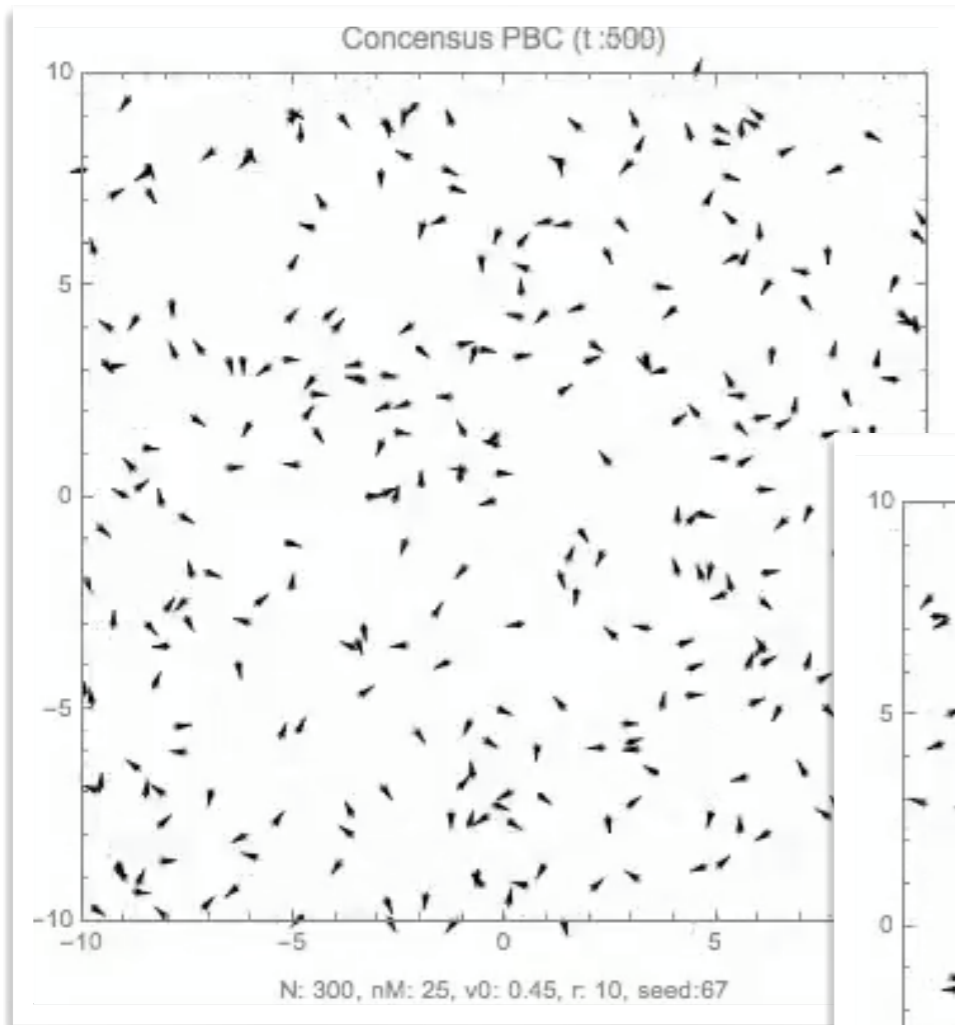
Flock Mates

. . .

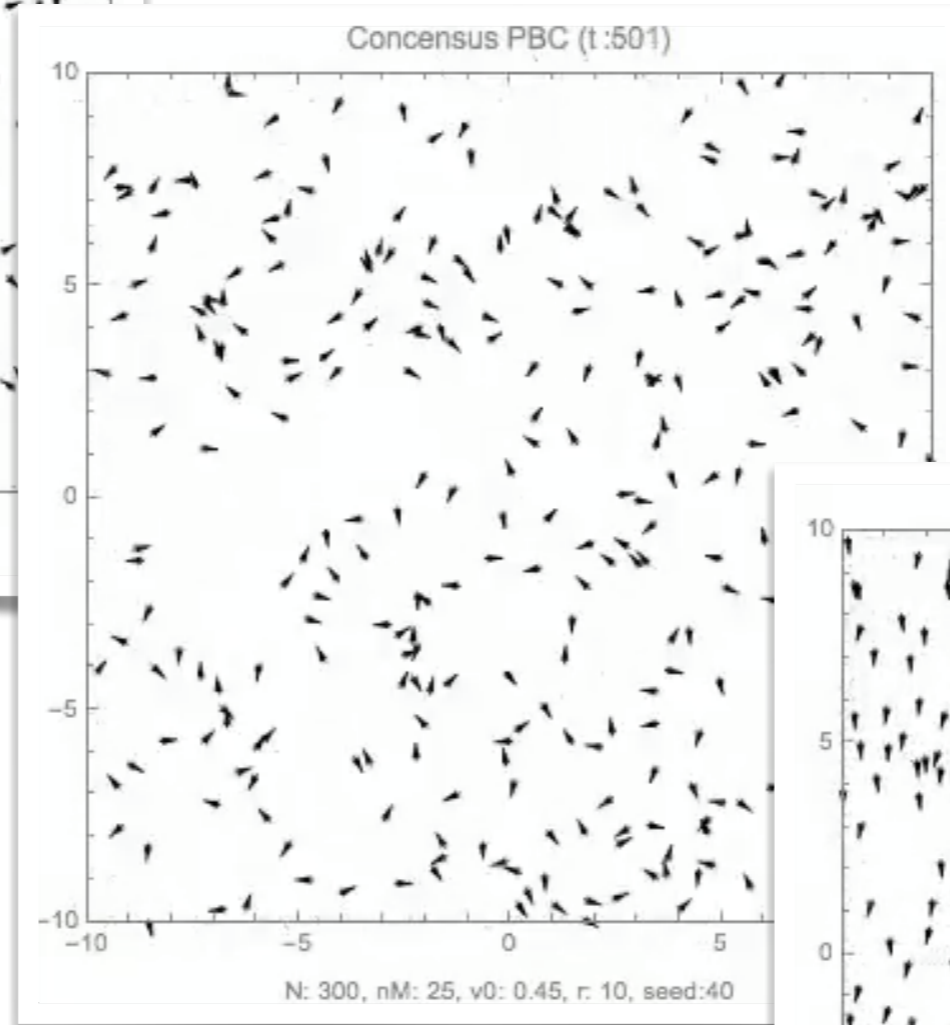
1 2 3 4 5 6 7 8 9 10 11 12 ... N-1 N

Democratic but not reciprocal, $1 \rightarrow 2$ does not imply $2 \rightarrow 1$

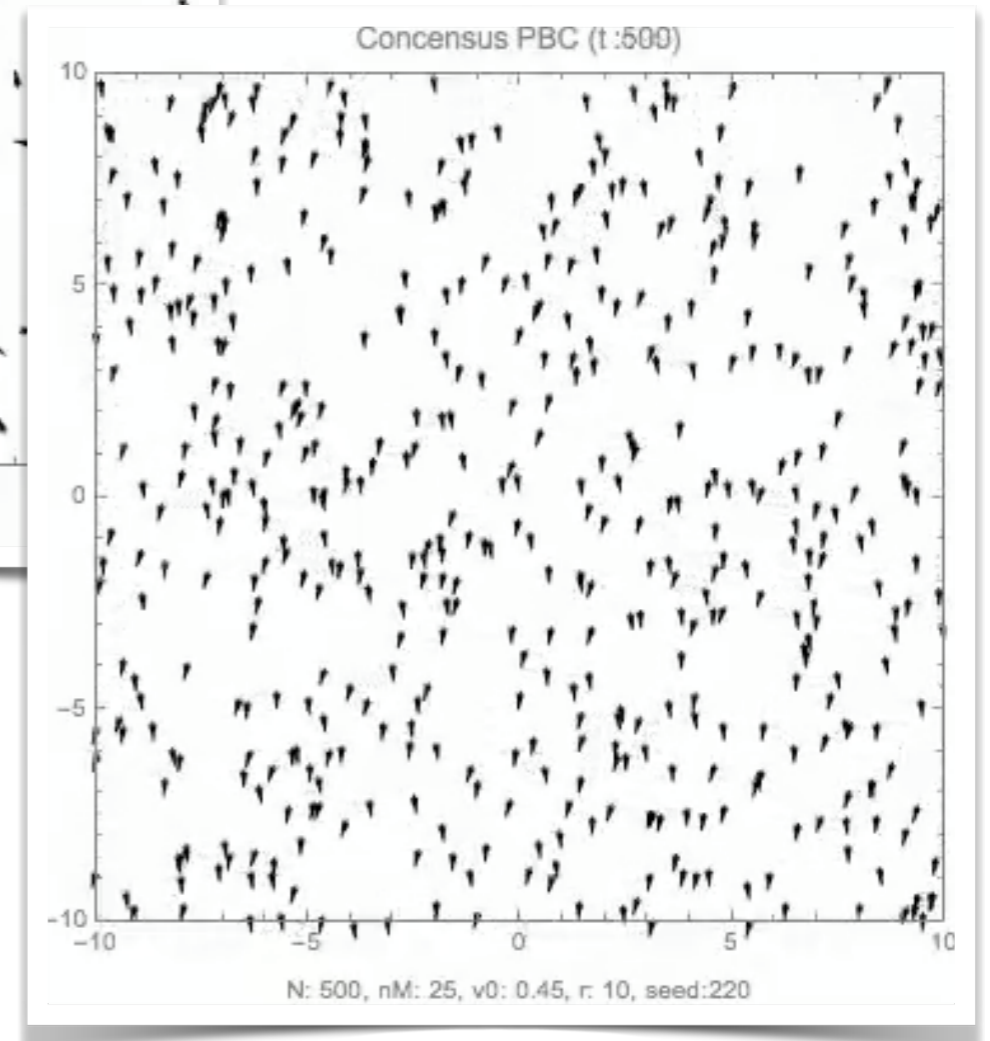
3 Unique Phases



Clockwise (-)



Counter-Clockwise (+)



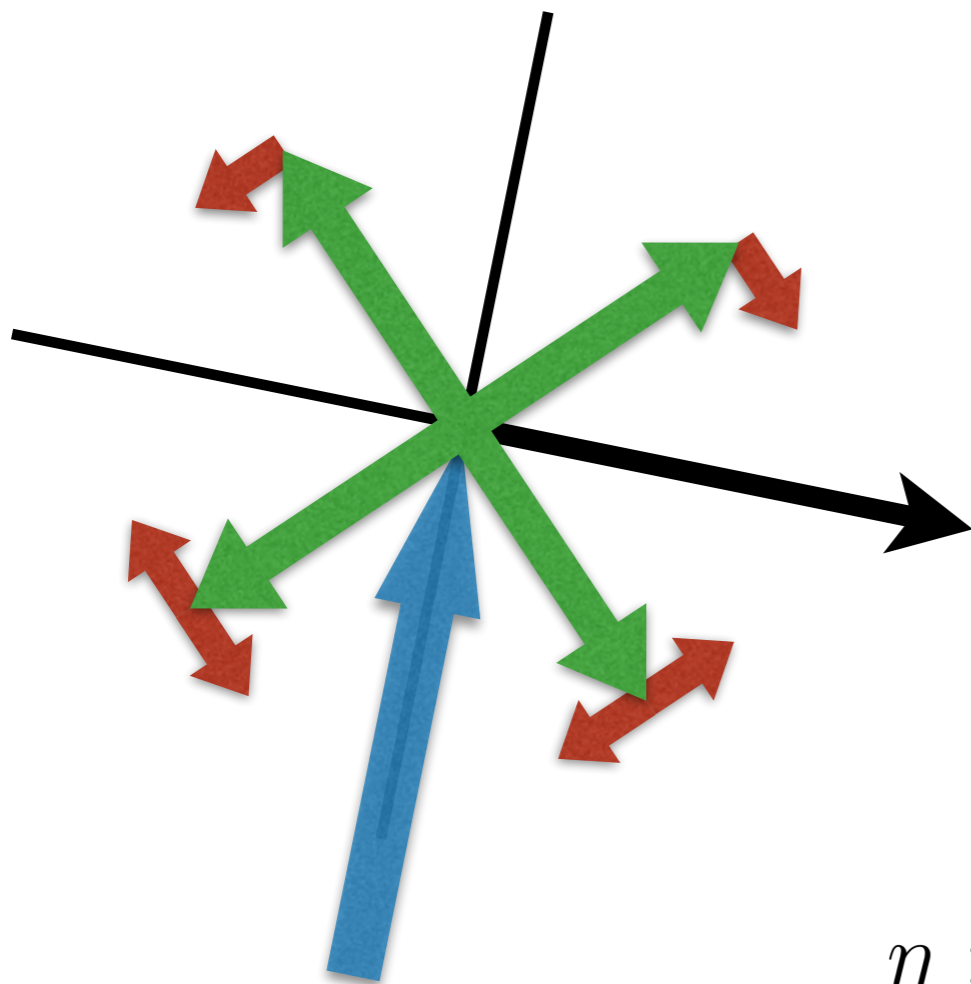
Aligned (0)

- **Periodic Boundary Conditions**

Add Frustration

$$\mathbf{If} : \mathbf{v}_i(t) \cdot \mathbf{r}_i(t) > 0 : \theta = \left(\frac{|\mathbf{r}_i(t)|}{L} \right)^p * \text{Sign}(\mathbf{v}_i(t) \cdot \mathbf{T}_i(t)) * |\eta|$$

$$\mathbf{Else} : \theta = \left(\frac{|\mathbf{r}_i(t)|}{L} \right)^p * \eta$$



$\mathbf{r}_i(t)$: position



$\mathbf{v}_i(t)$: velocity



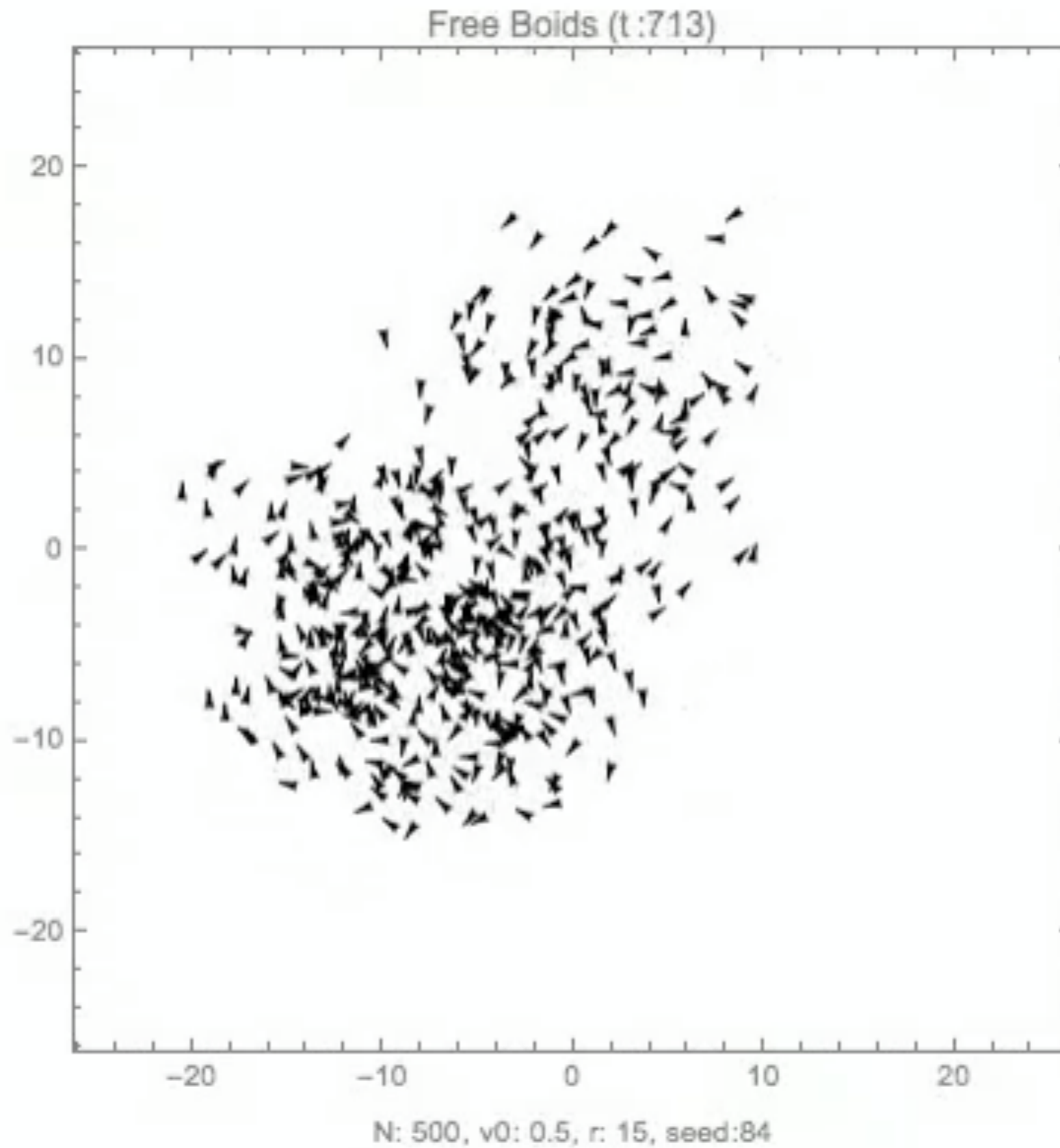
θ : frustration



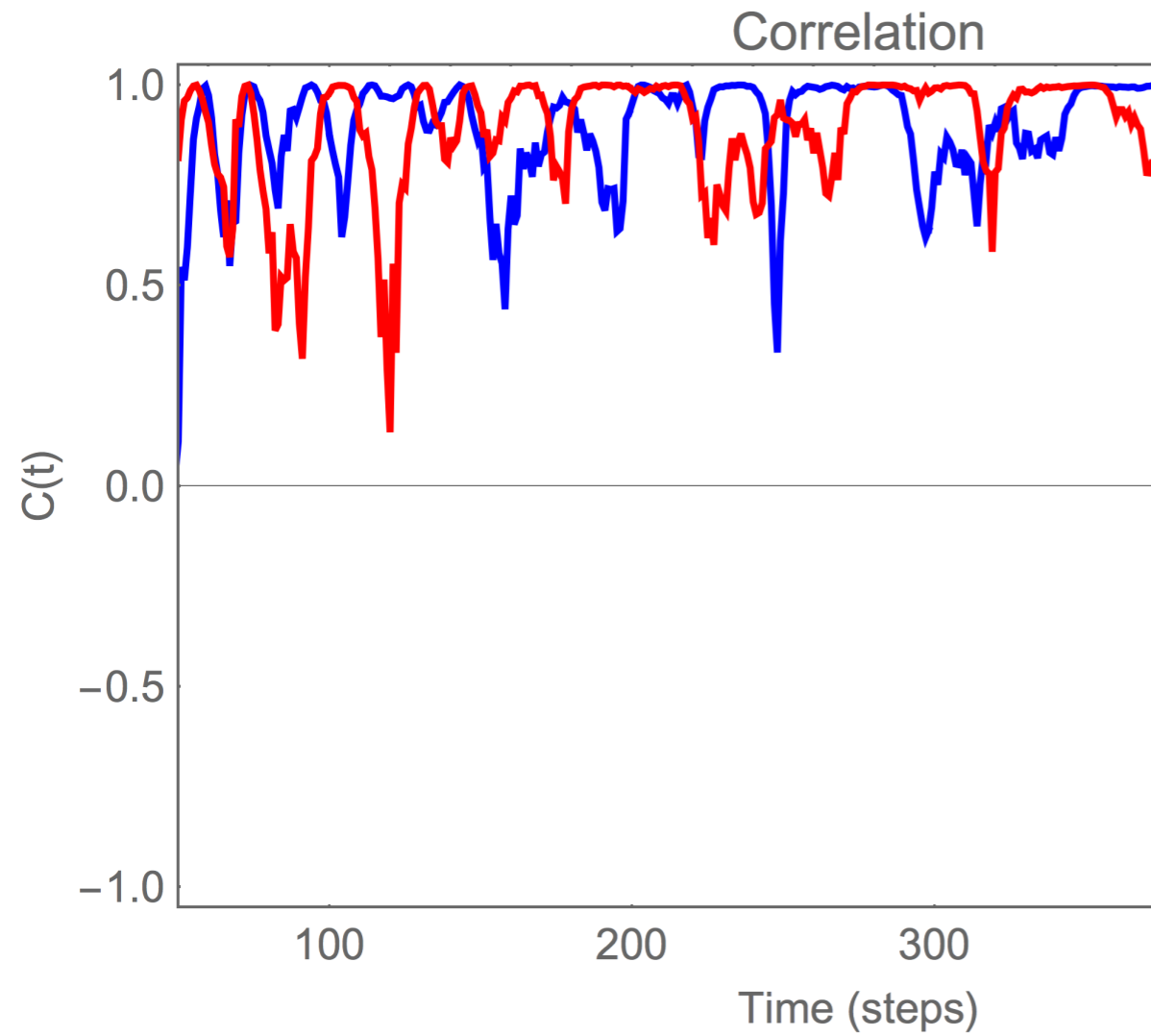
$\mathbf{T}_i(t)$: tangential

η : normally distributed random number

Add Frustration

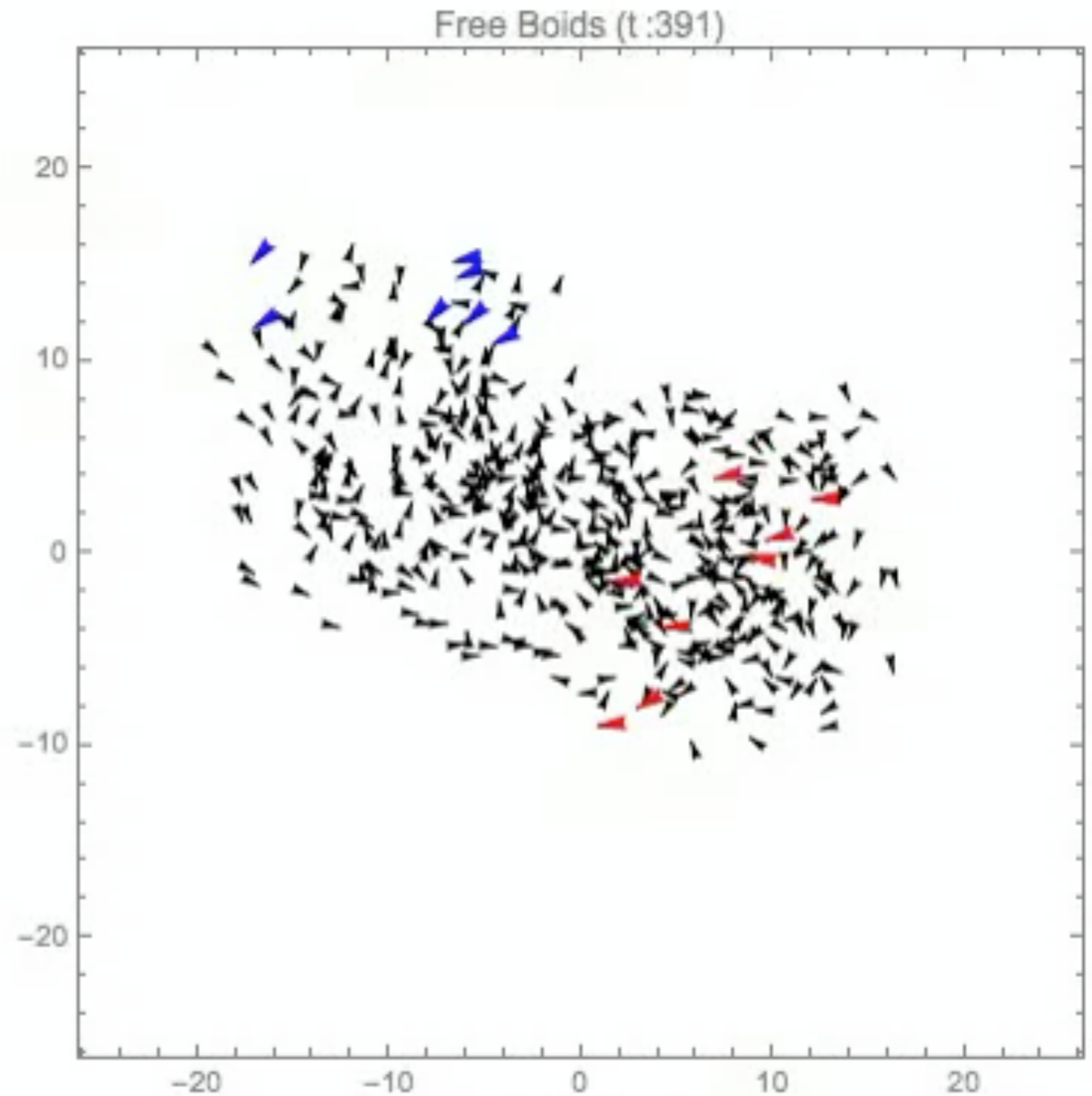


Order Parameters and Phase Transitions



— Boid 1
— Boid 250

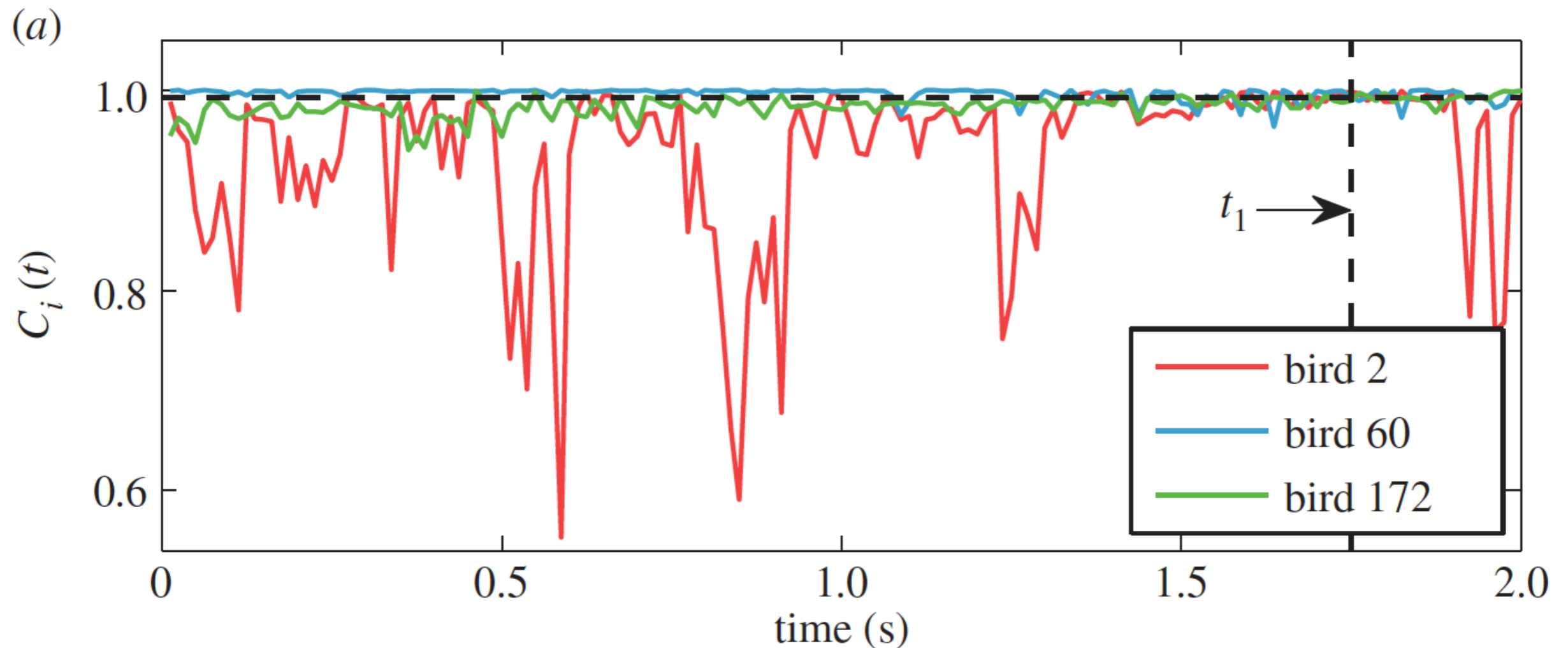
Time: 1



Directional Correlation by Observation

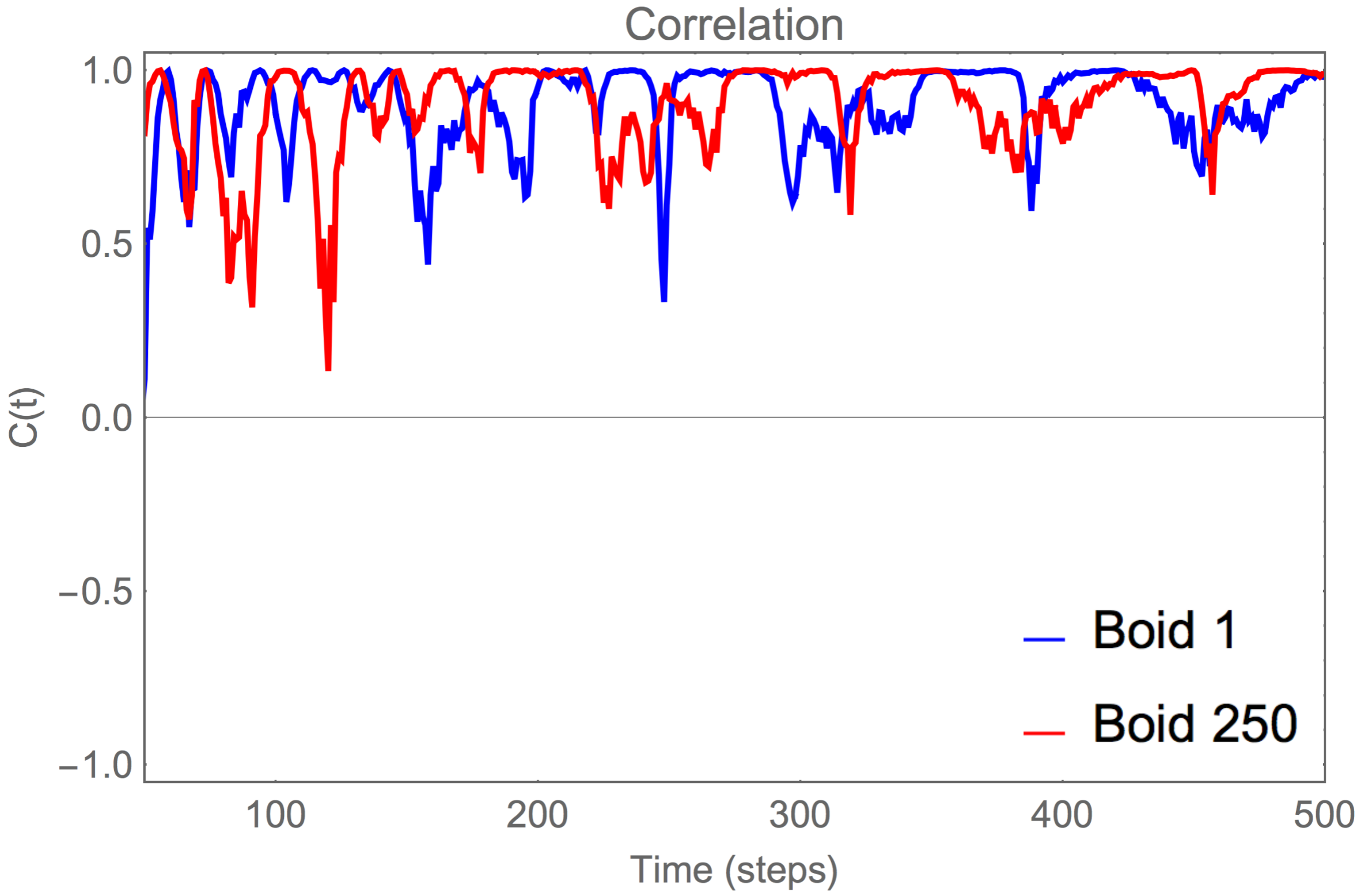
"Emergence of collective changes in travel direction of starling flocks from individual birds' fluctuations"

- A. Alessandro, A. Cavagna, et al.



Attanasi, Alessandro, Andrea Cavagna, Lorenzo Del Castello, Irene Giardina, Asja Jelic, Stefania Melillo, Leonardo Parisi, Oliver Pohl, Edward Shen, and Massimiliano Viale. "Emergence of collective changes in travel direction of starling flocks from individual birds' fluctuations." *Journal of The Royal Society Interface* 12, no. 108 (2015): 20150319.

Directional Correlation



Summary



- Simple model, rich behavior
- Velocity Averaging causes phases
- Frustration causes chaos
- A delicate balance between the two results in emergence.
- The results are similar to observations.



Order Parameters

Alignment Order Parameter

$$\langle v(t) \rangle = \frac{1}{N v_0} \left| \sum_{i=1}^N \vec{v}_i(t) \right|$$

1 if all boids are aligned
0 if random or rotating

Rotational Order Parameter

$$L(t) = \frac{1}{N} \sum_{i=1}^N \frac{1}{z} \sum_{t=1}^z \frac{v_i(t) \wedge v_i(t+1)}{v_0^2}$$

Clockwise (-)
Counter-
Clockwise (+)

N is the total number of boids

Order Parameters

Velocity Correlation

$$C_i(t) = \frac{\mathbf{v}_i(t)}{v_i(t)} \cdot \frac{\mathbf{V}_n(t)}{V_n(t)} \quad \begin{array}{l} 1 : \text{strongly correlated} \\ -1 : \text{anti-correlated} \end{array}$$

where

$$\mathbf{V}_n(t) = \sum_{i=1}^n \mathbf{v}_i(t)$$

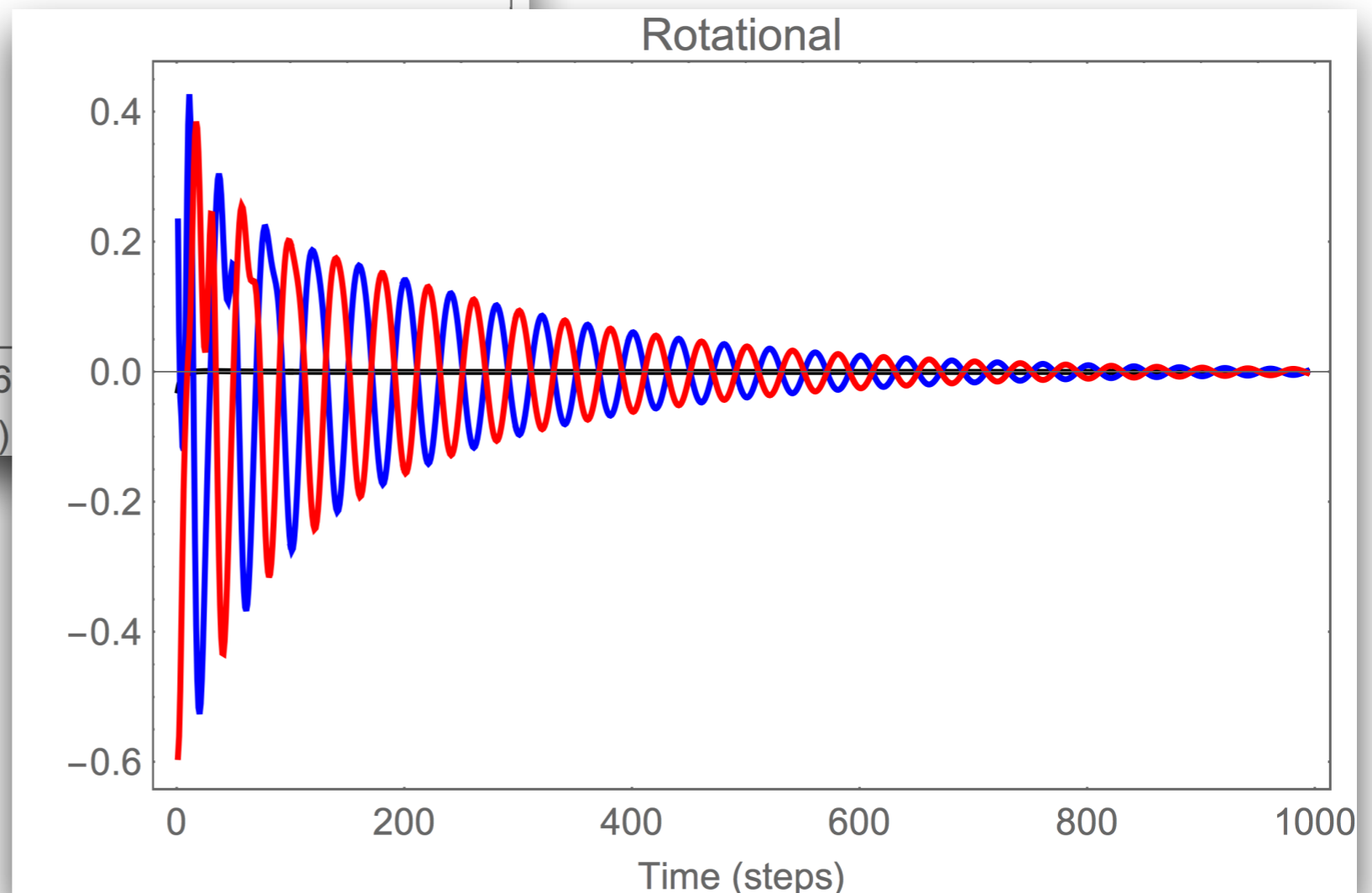
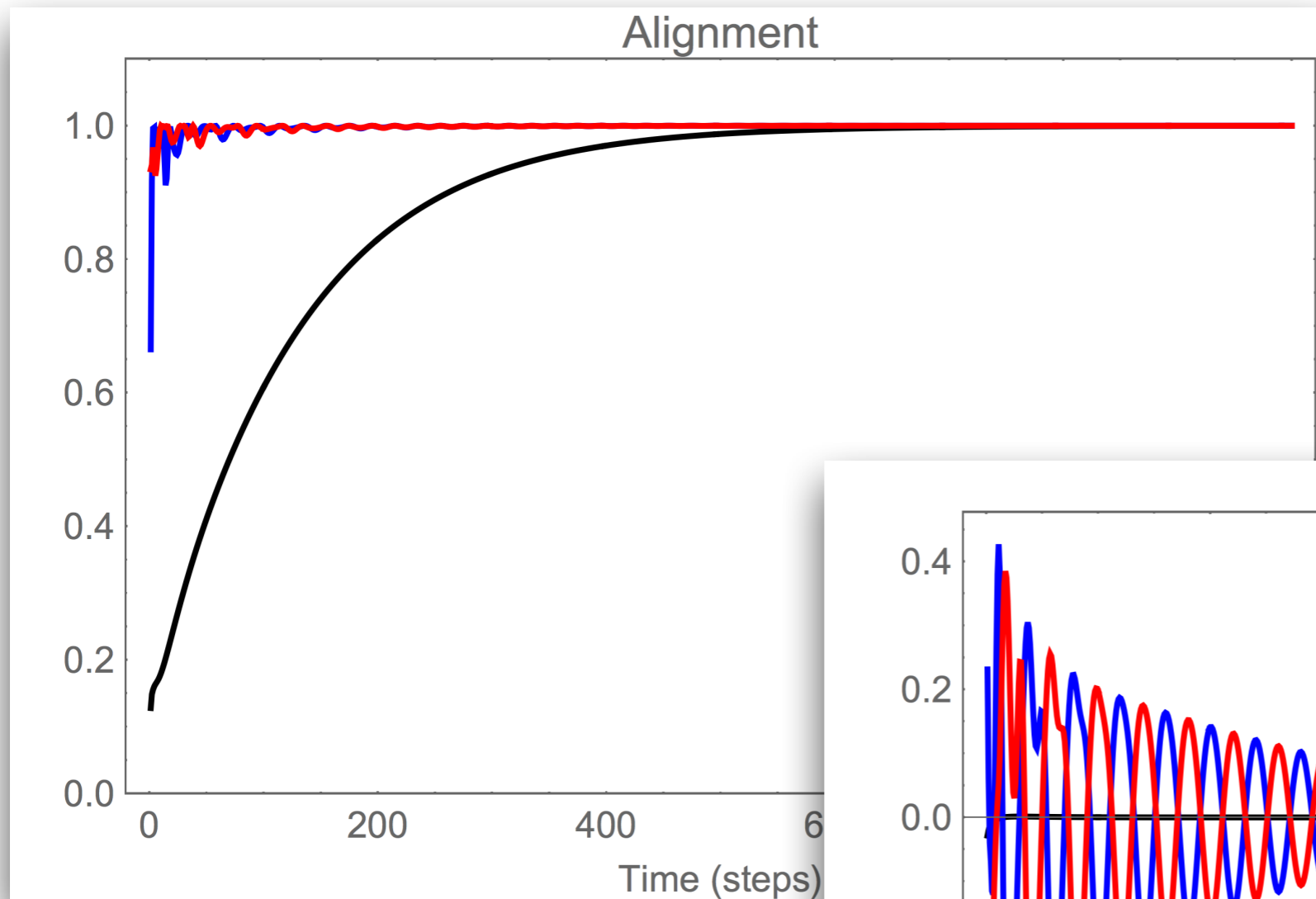
Order Parameters

Average Distance between flockmates

$$\mathbf{R}(t) = \frac{1}{n} \sum_{i=1}^n |\mathbf{r}(t) - \mathbf{r}_i(t)|$$

Periodic Boundaries

Aligned



N: 500

n: 25

v0: 0.45

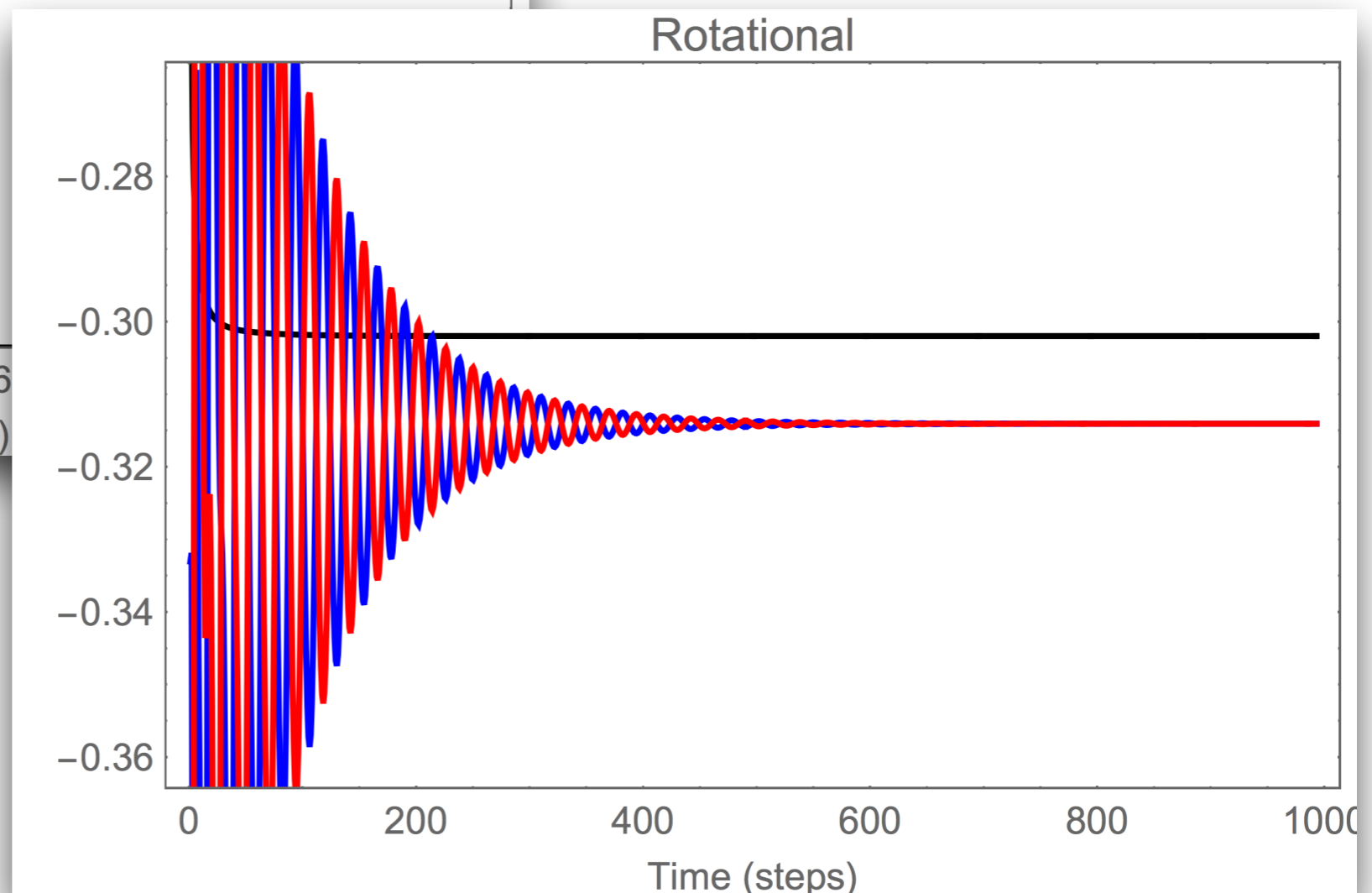
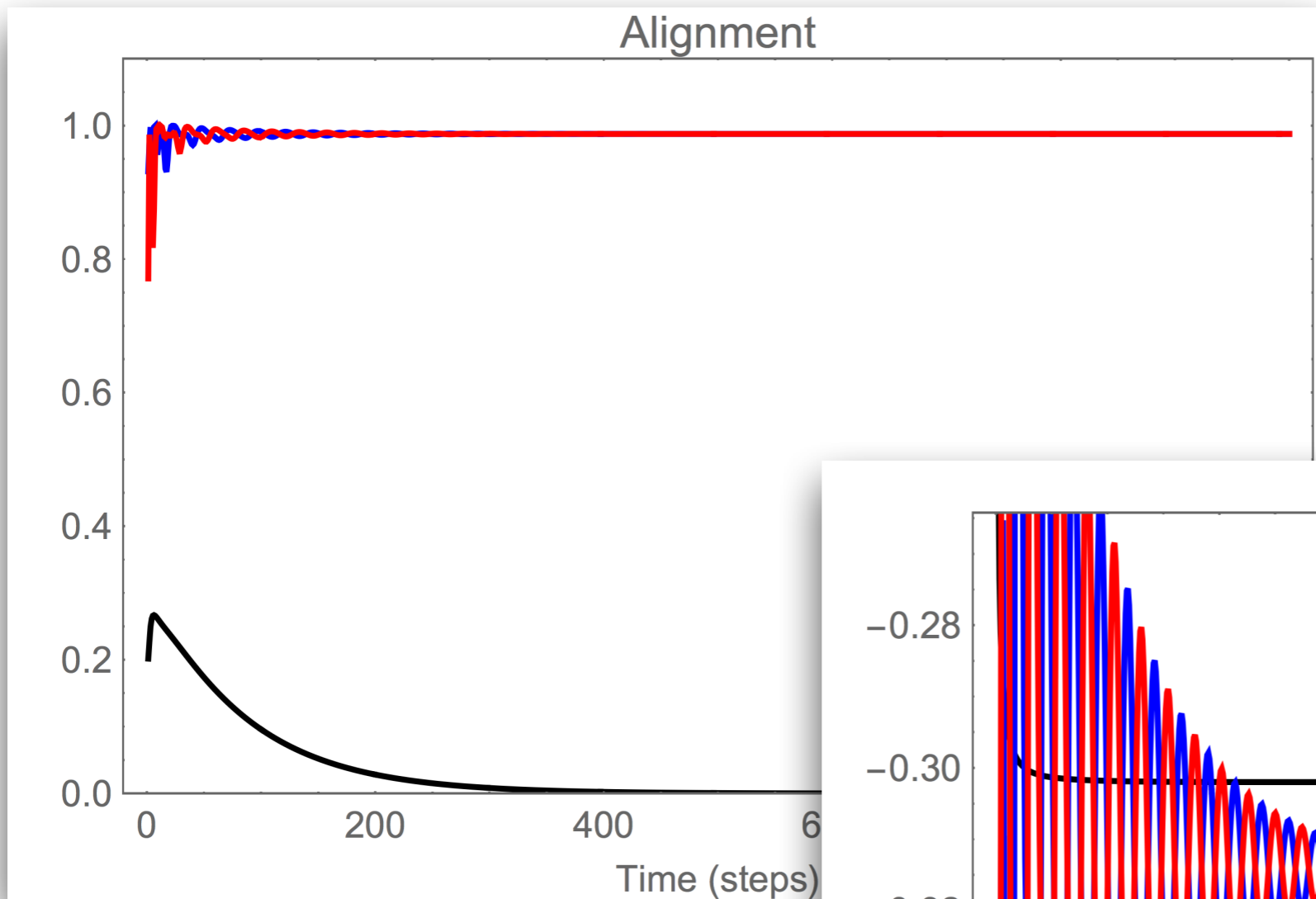
r: 10

Seed: 220

BC: PBC

Periodic Boundaries

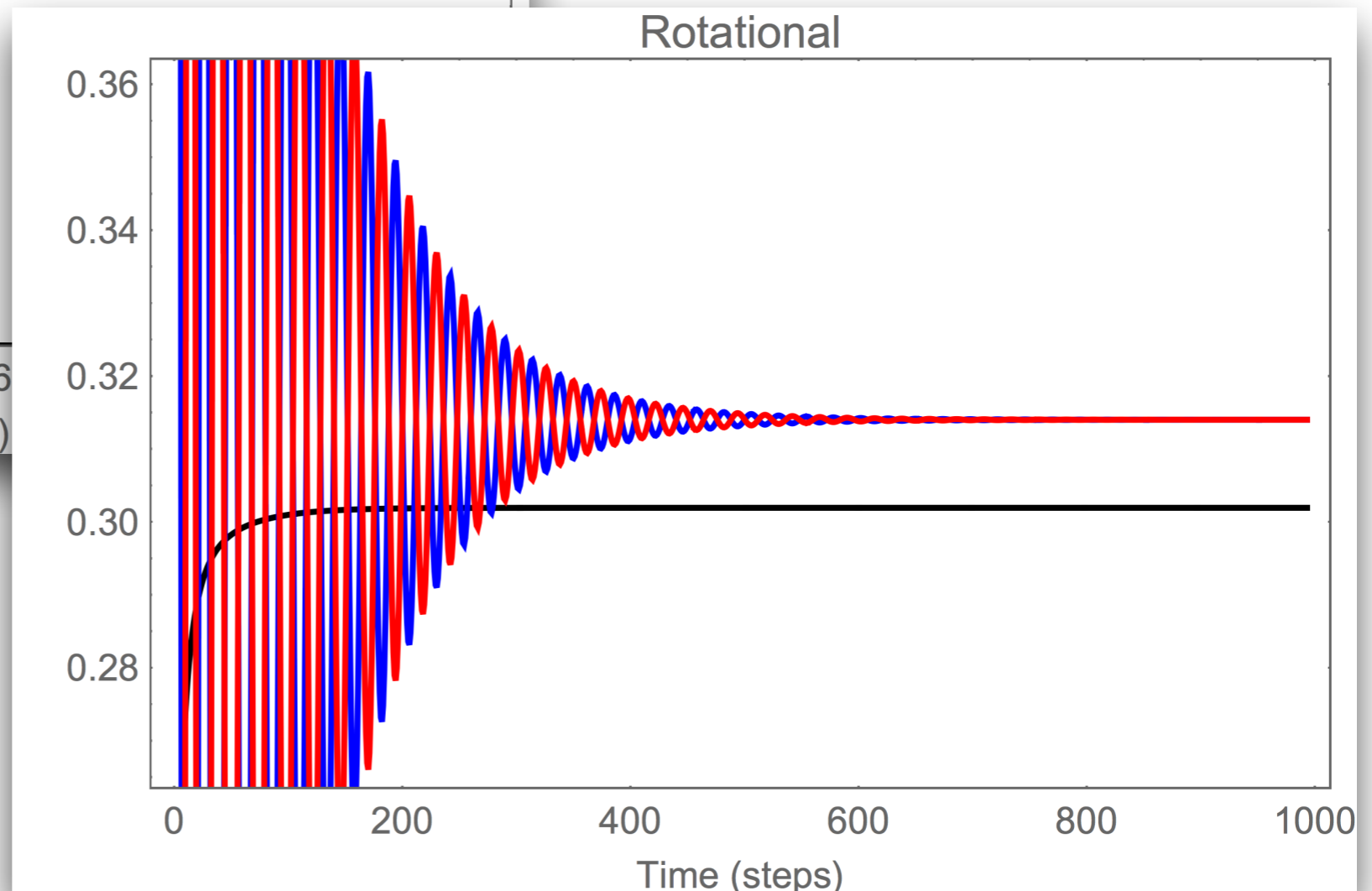
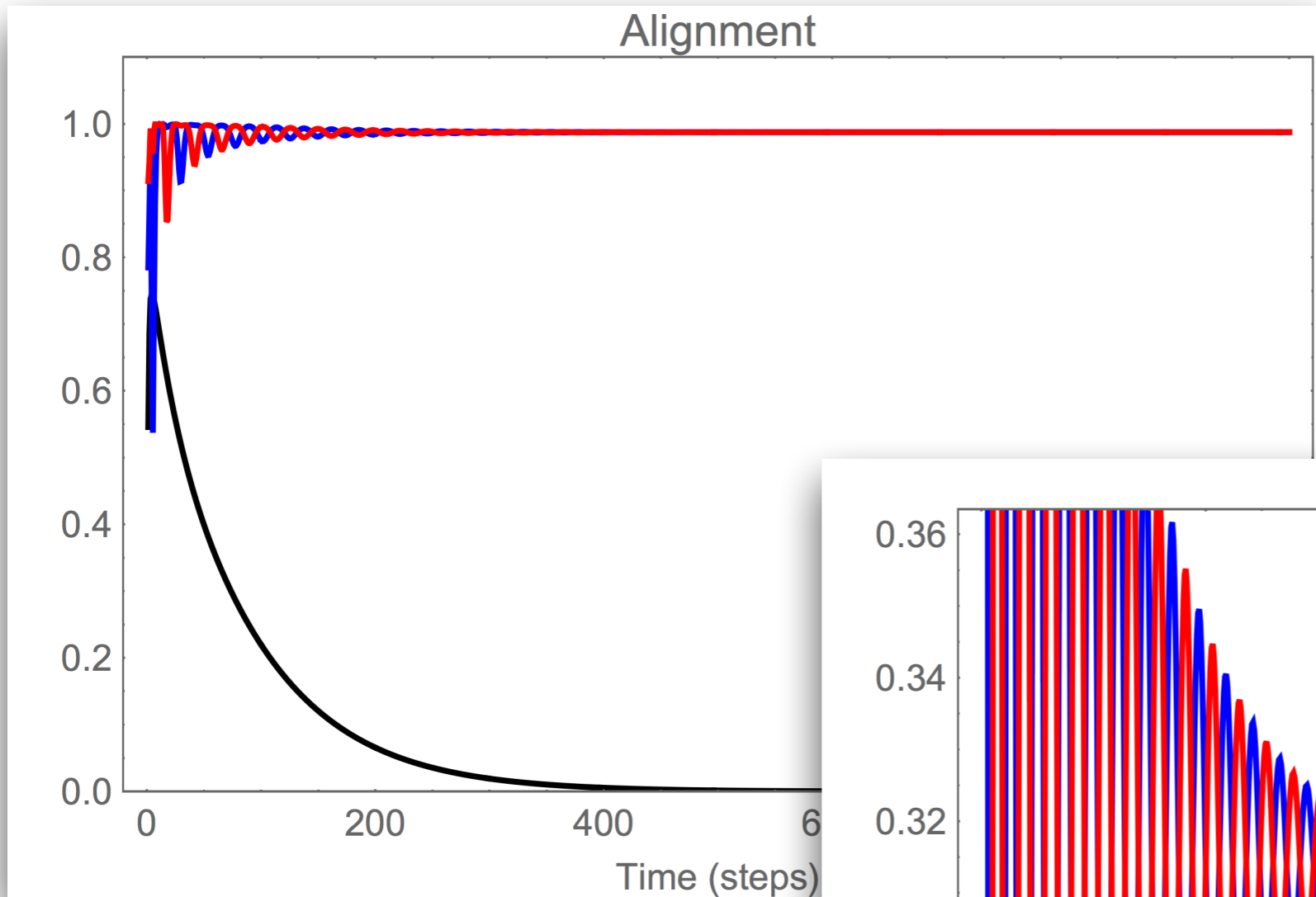
Clockwise



N: 300
n: 25
v0: 0.45
r: 10
Seed: 67
BC: PBC

Periodic Boundaries

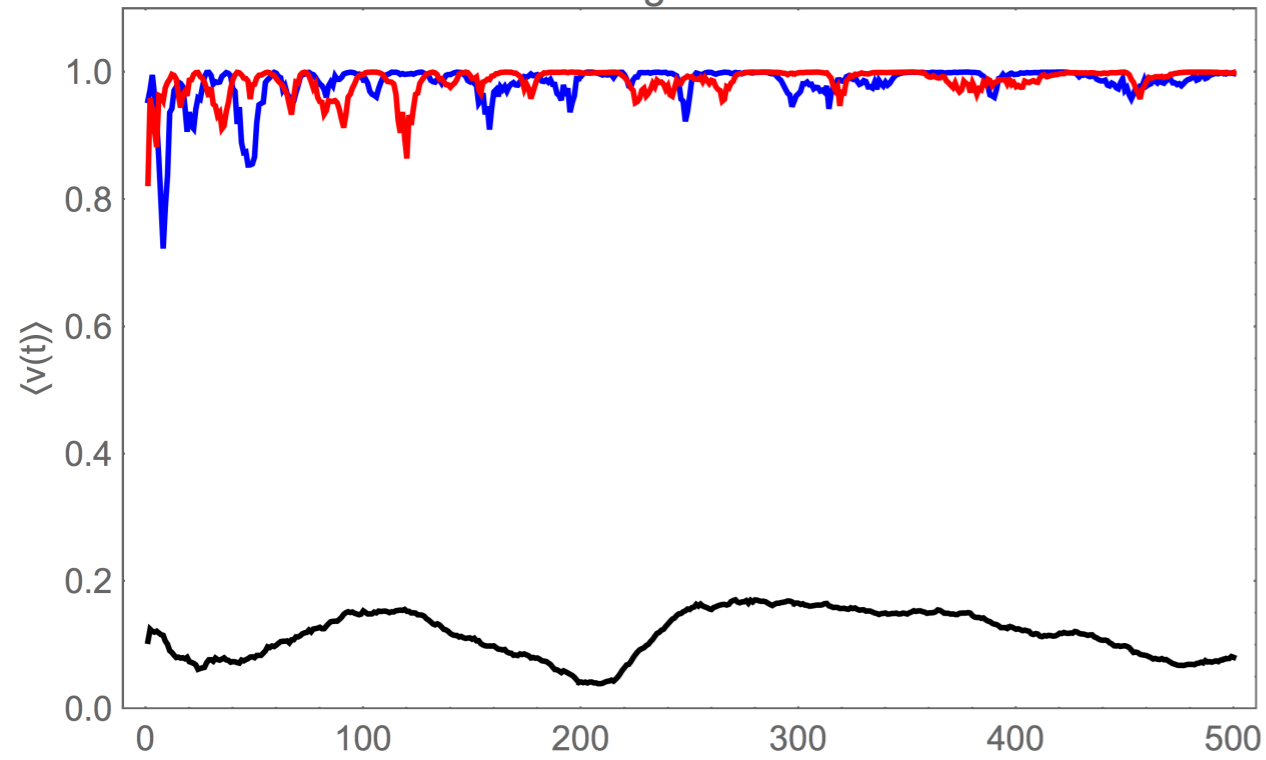
Counter-
Clockwise



N: 300
n: 25
v0: 0.45
r: 10
Seed: 40
BC: PBC

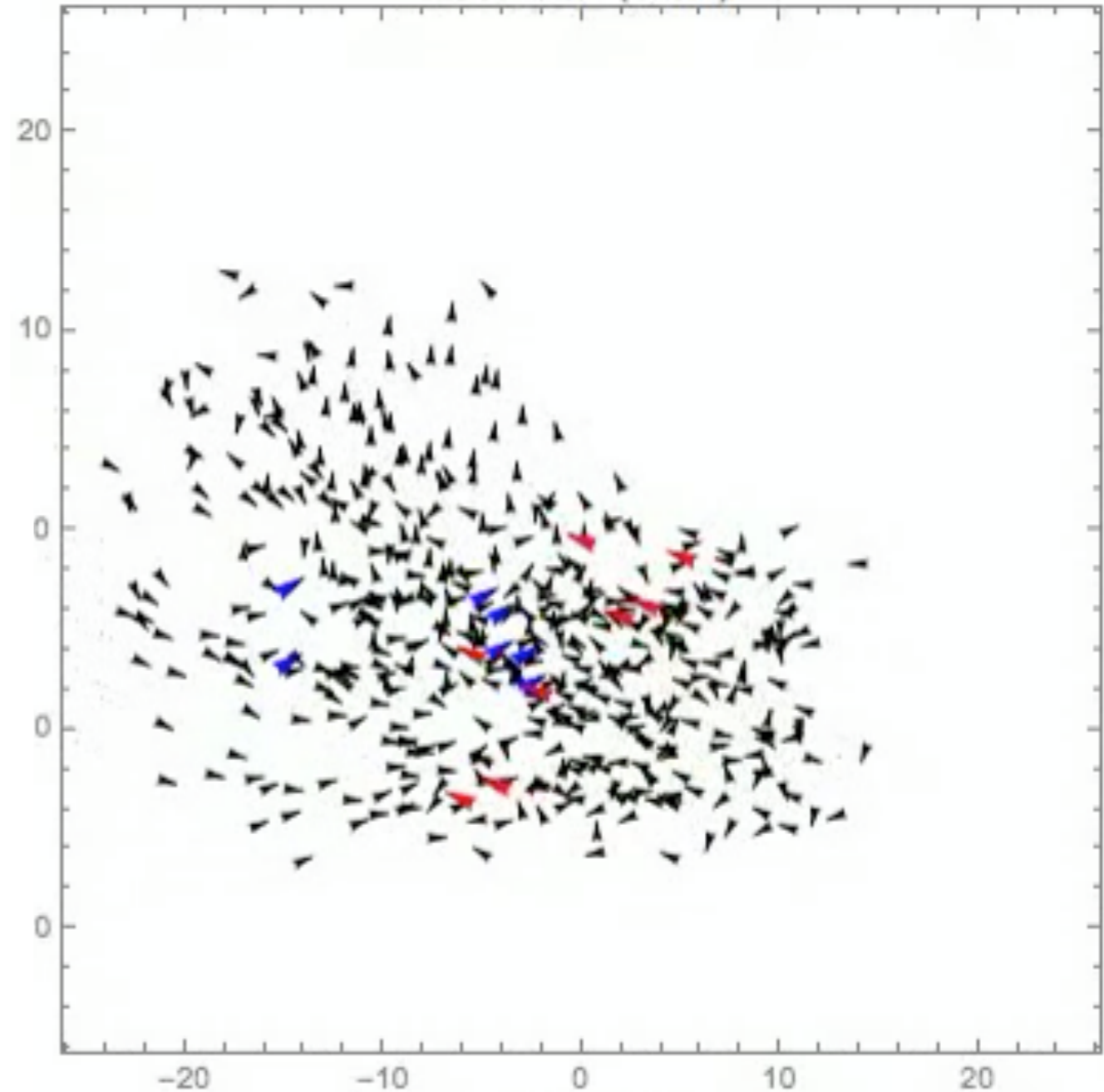
Order Parameters and Phase Transitions

Alignment

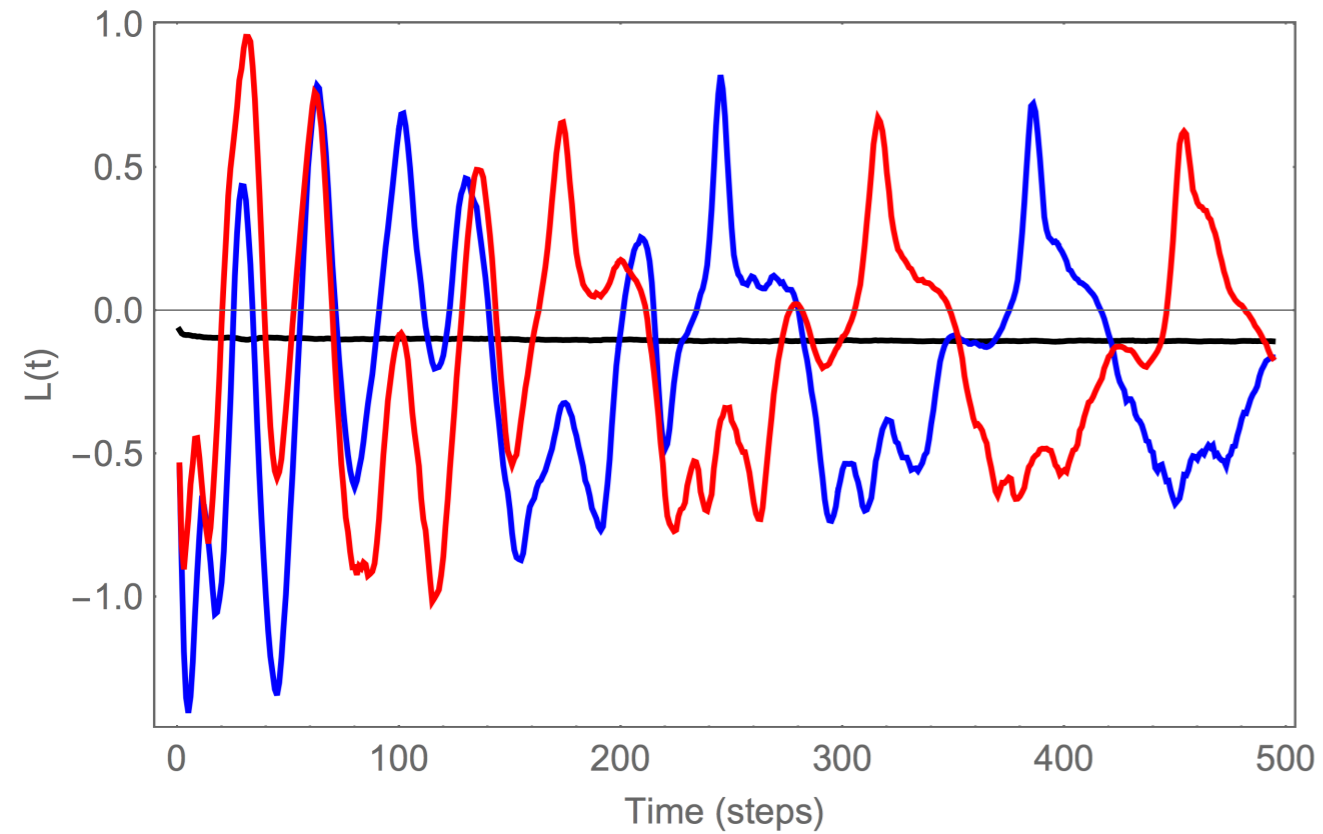


Time: 1

Free Boids (t :283)



Rotational

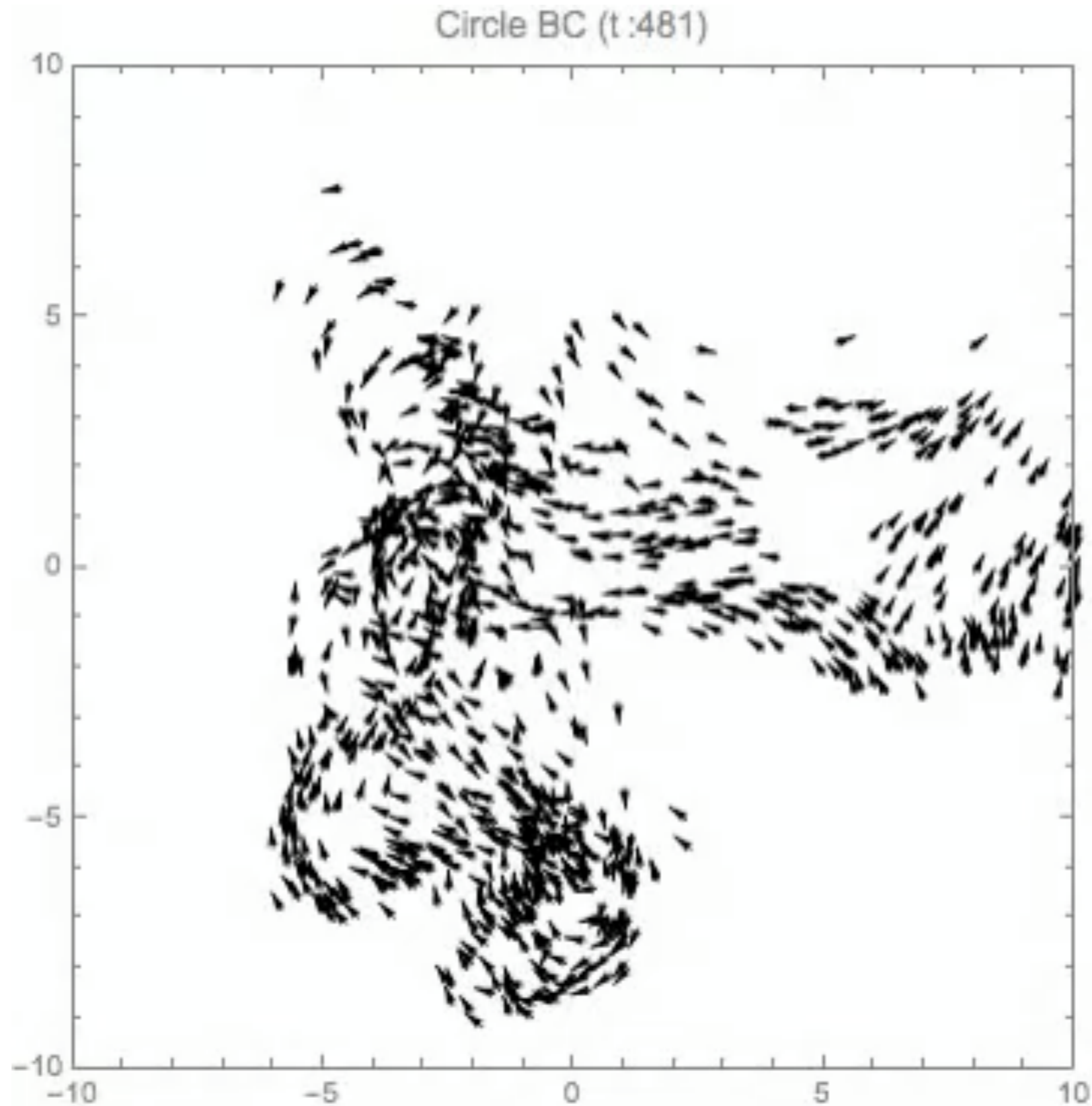


N: 500, v0: 0.5, r: 15, seed:84

Previous Work

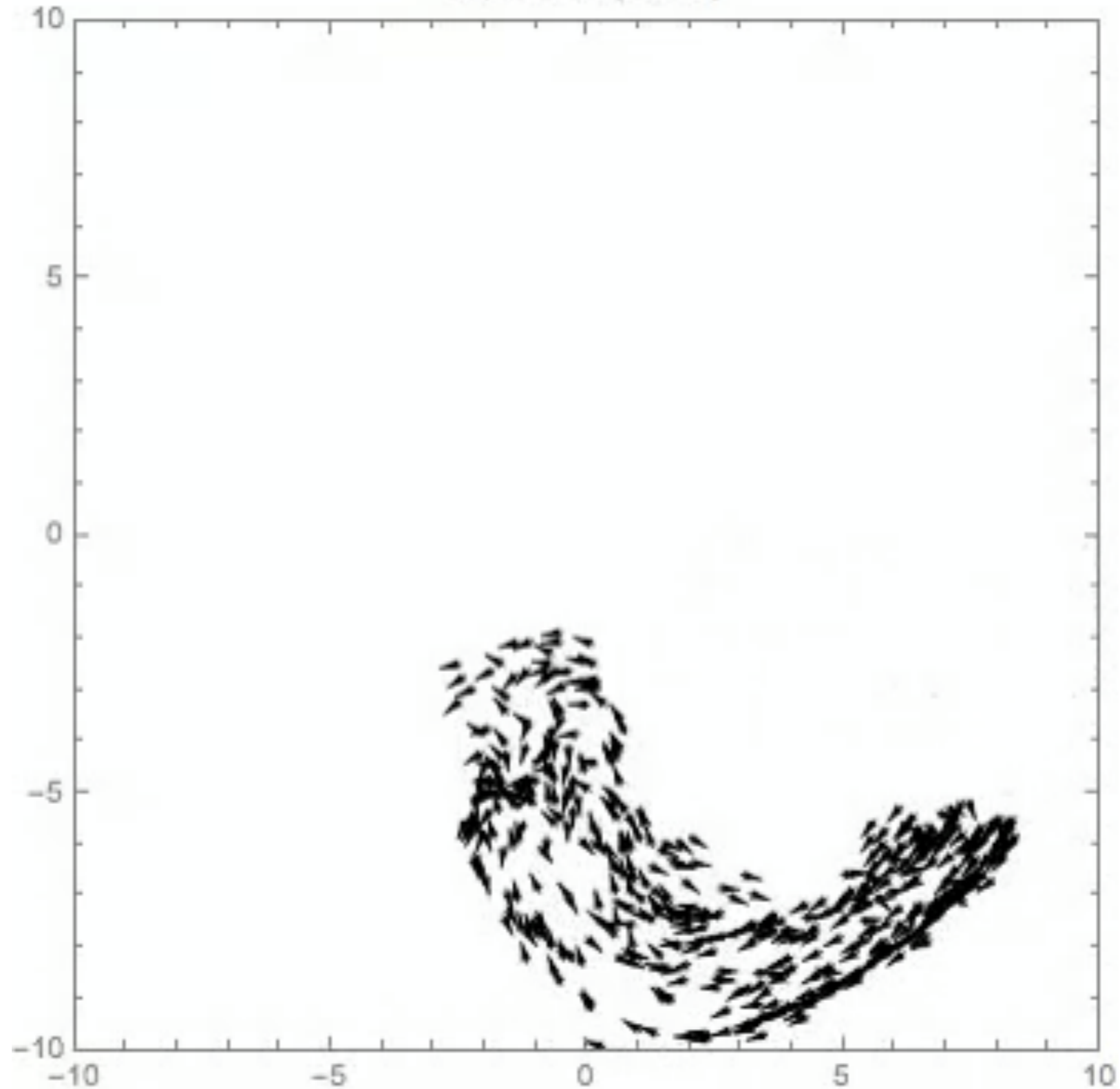
- Implemented frustration in the form of Boundary Conditions
 - U-turns and specular turns
 - "Soft basin" and "Rigid basin"
 - Square and Circular basins

Rigid Boundary: U-turn



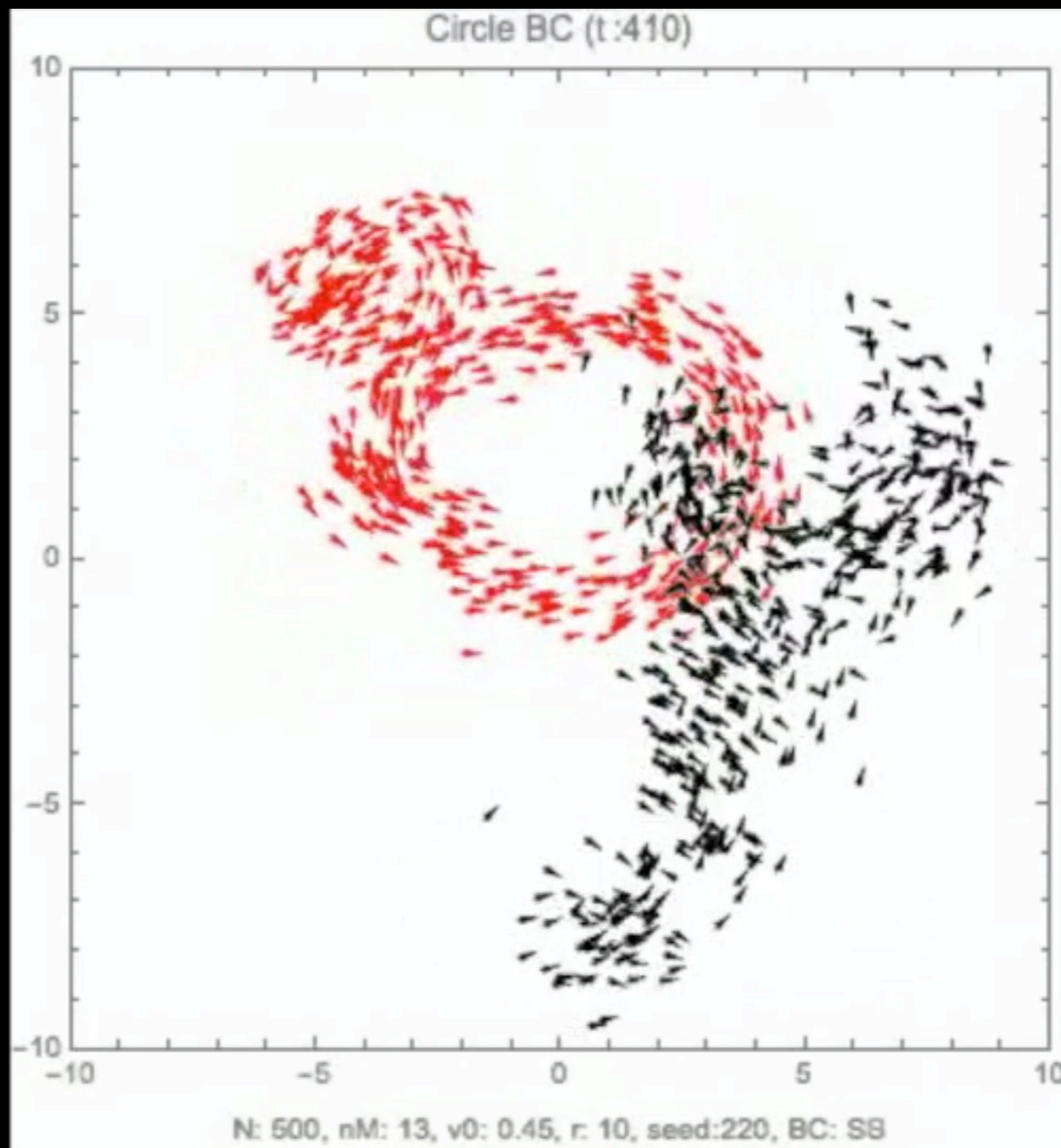
Rigid Boundary: Specular

Circle BC (t:742)



Comparing Boundary Conditions

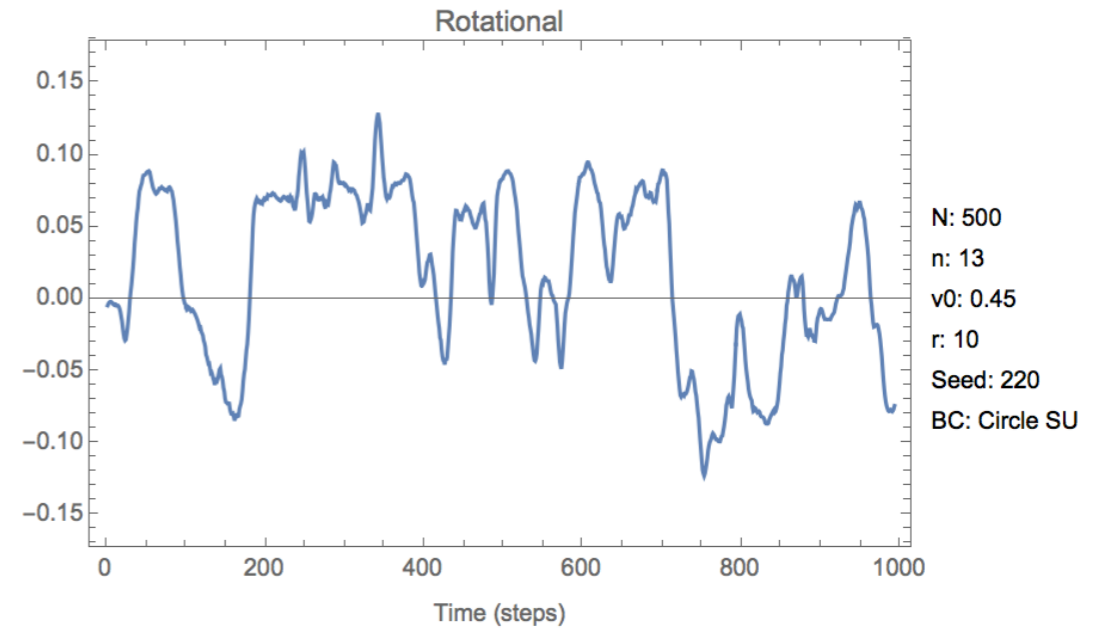
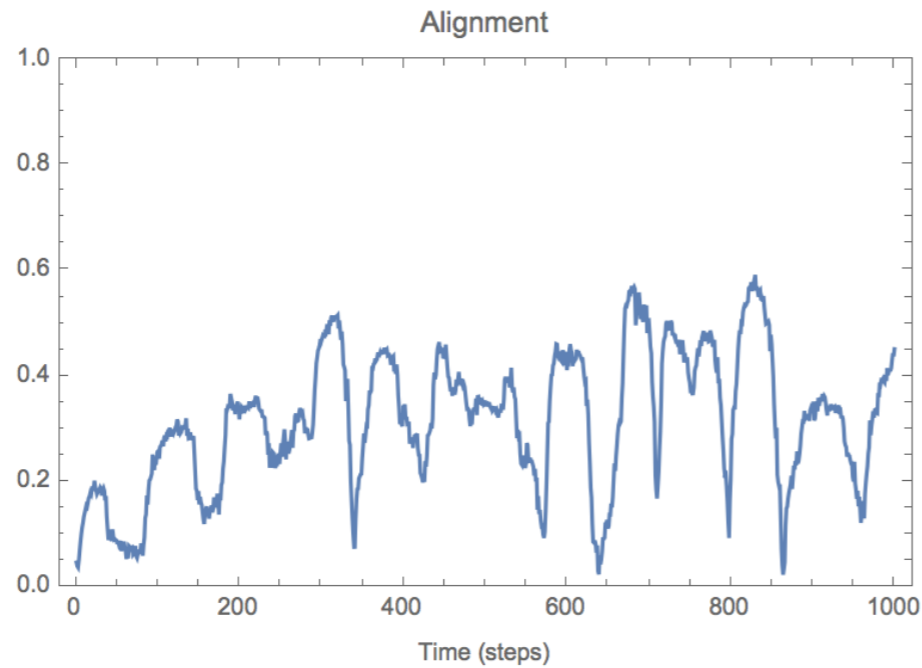
U-Turn



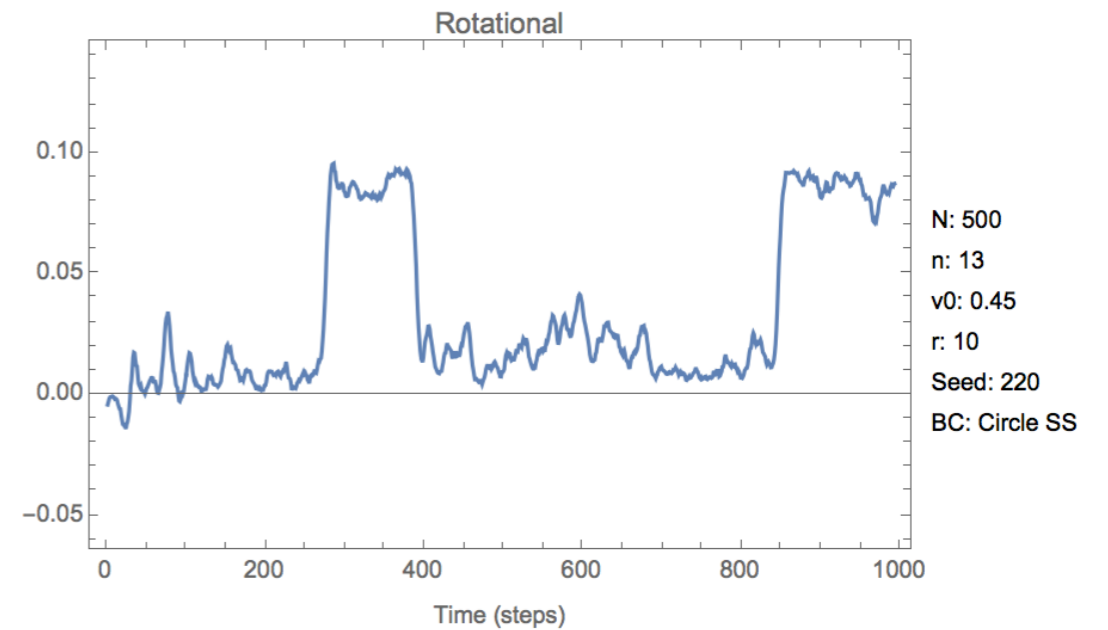
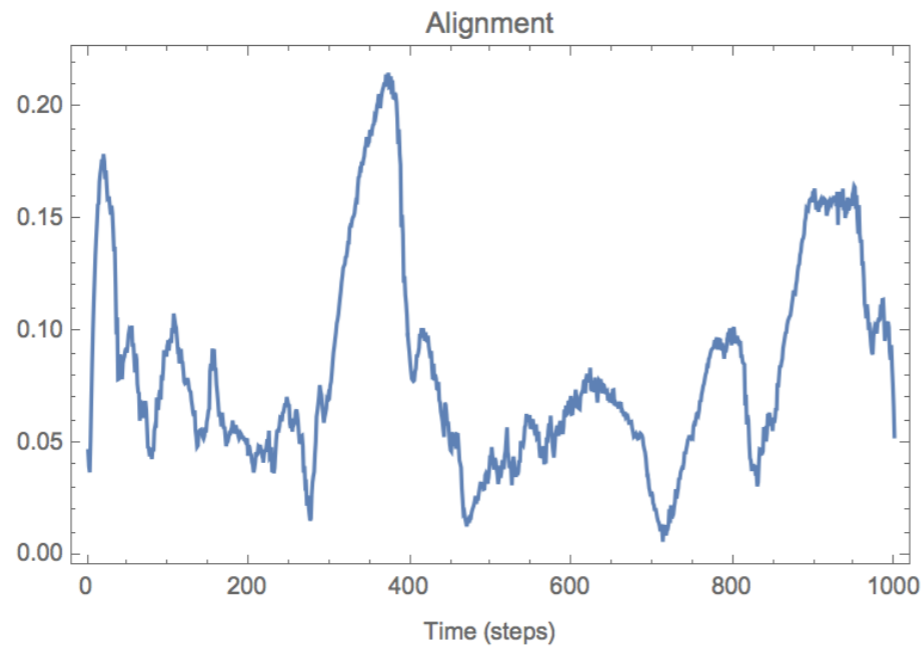
Specular

Comparing Boundary Conditions

**Circle
U-Turn**

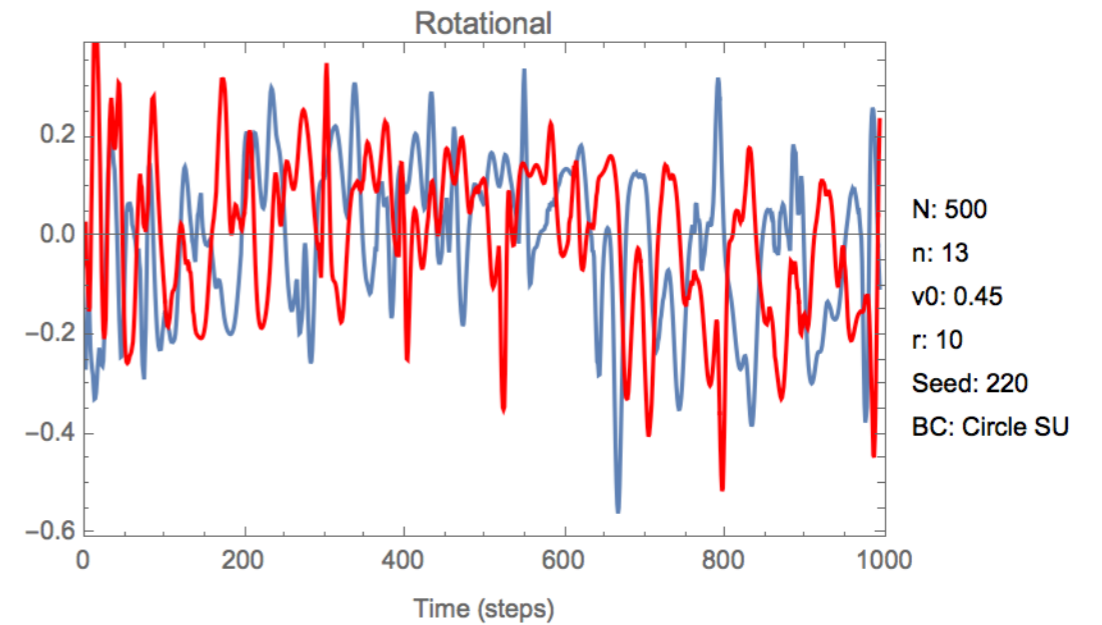
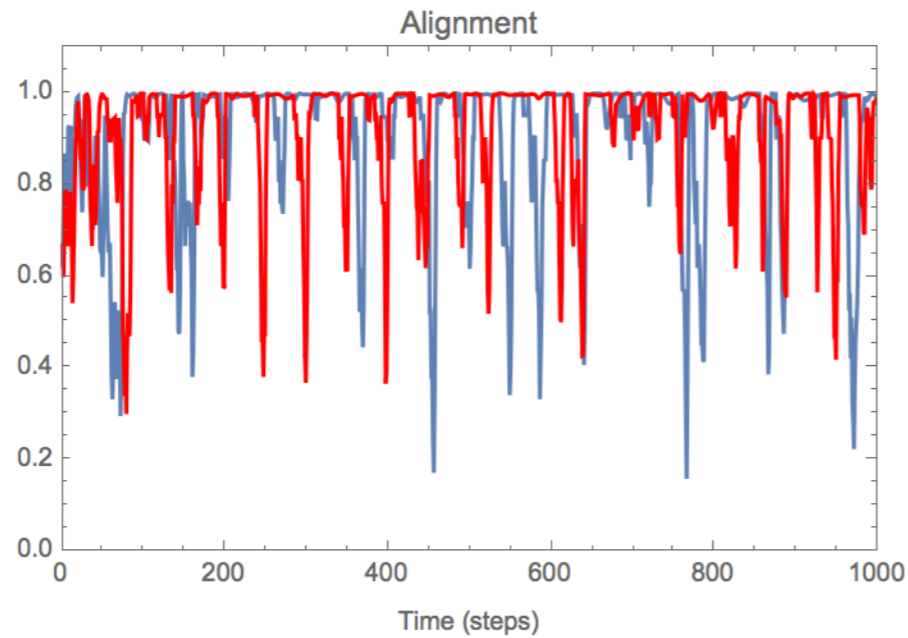


**Circle
Specular**

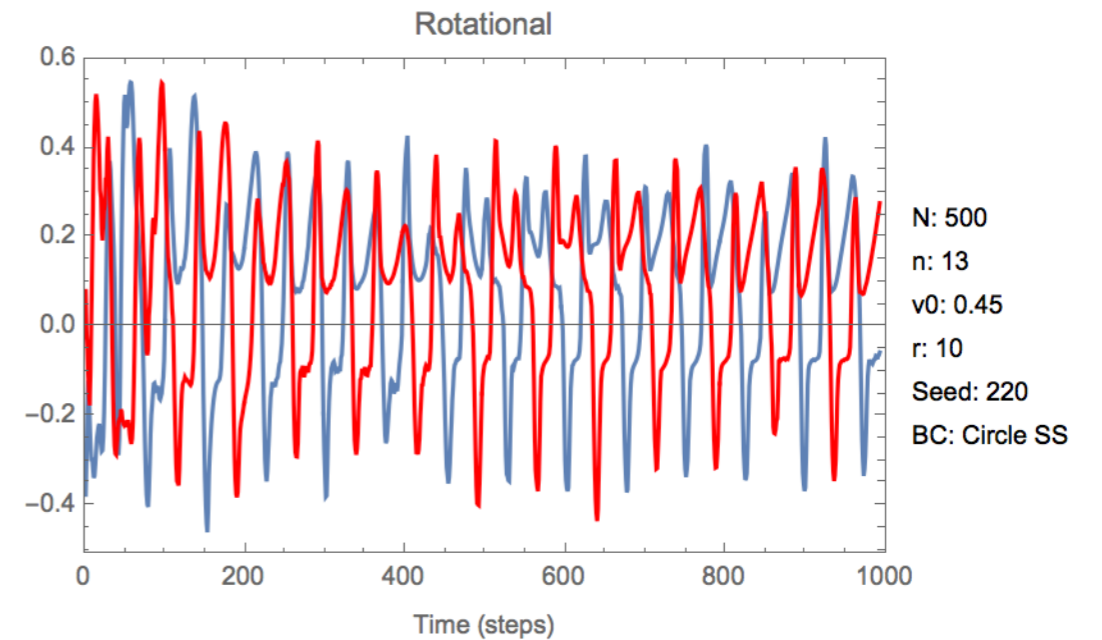
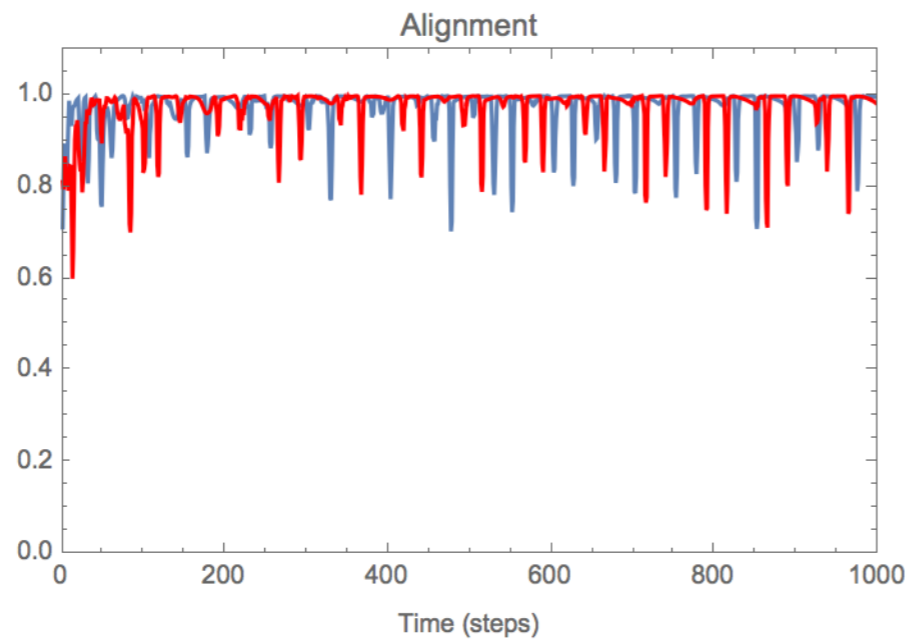


Comparing Boundary Conditions

**Circle
U-Turn**

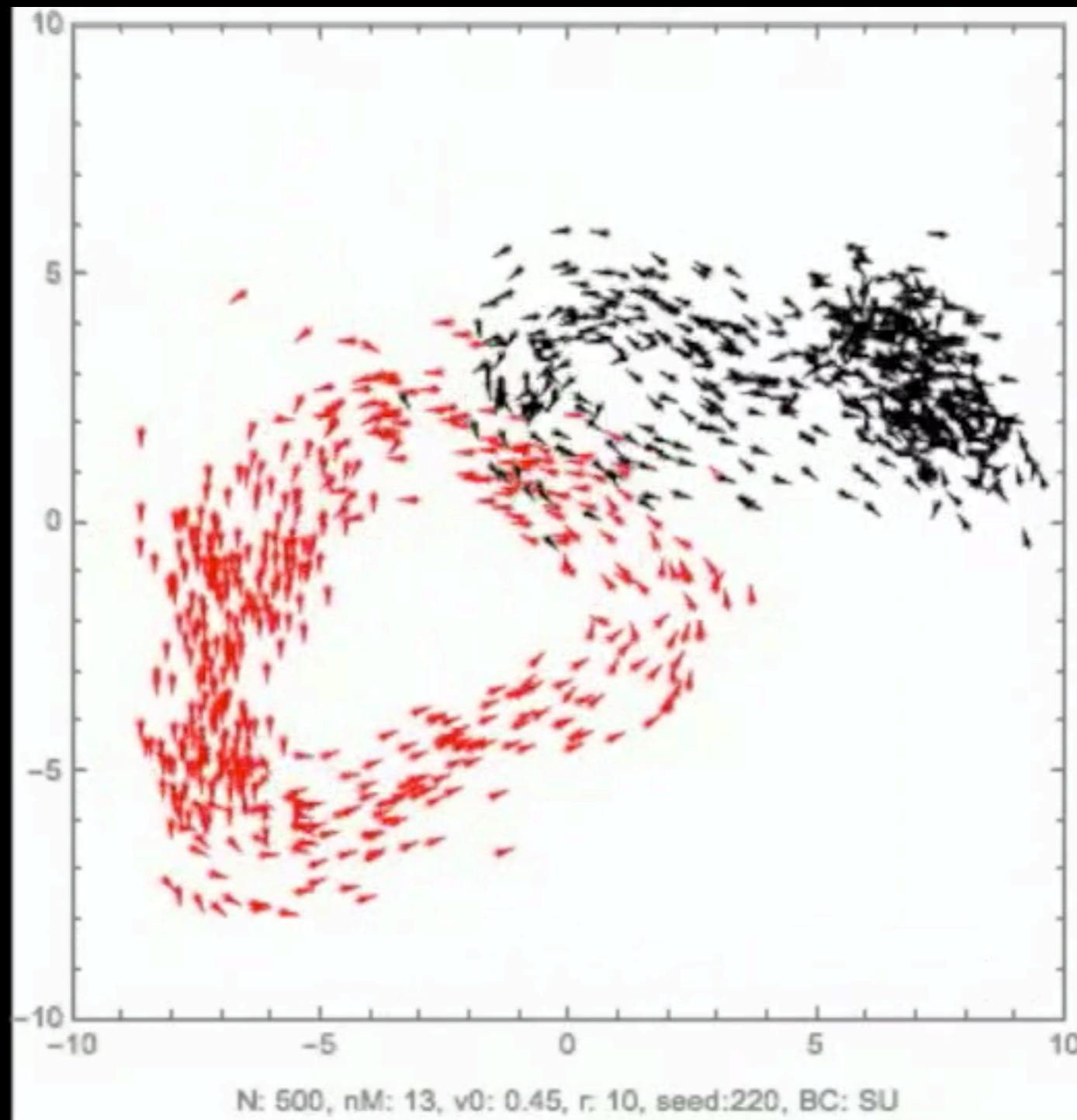


**Circle
Specular**



Comparing Boundary Conditions

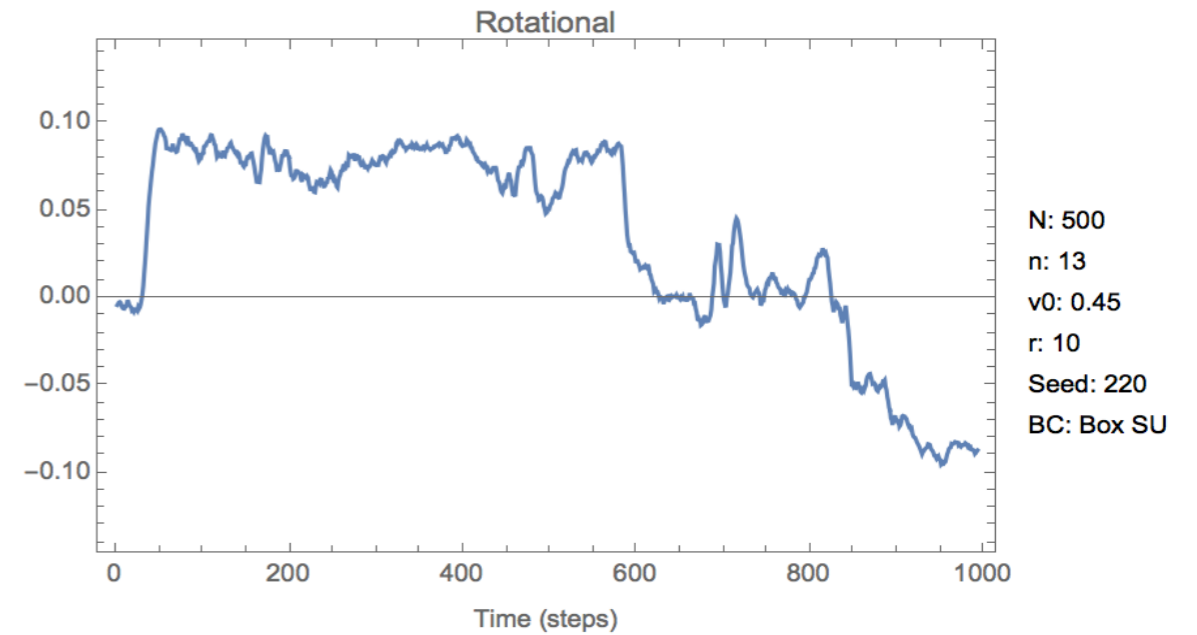
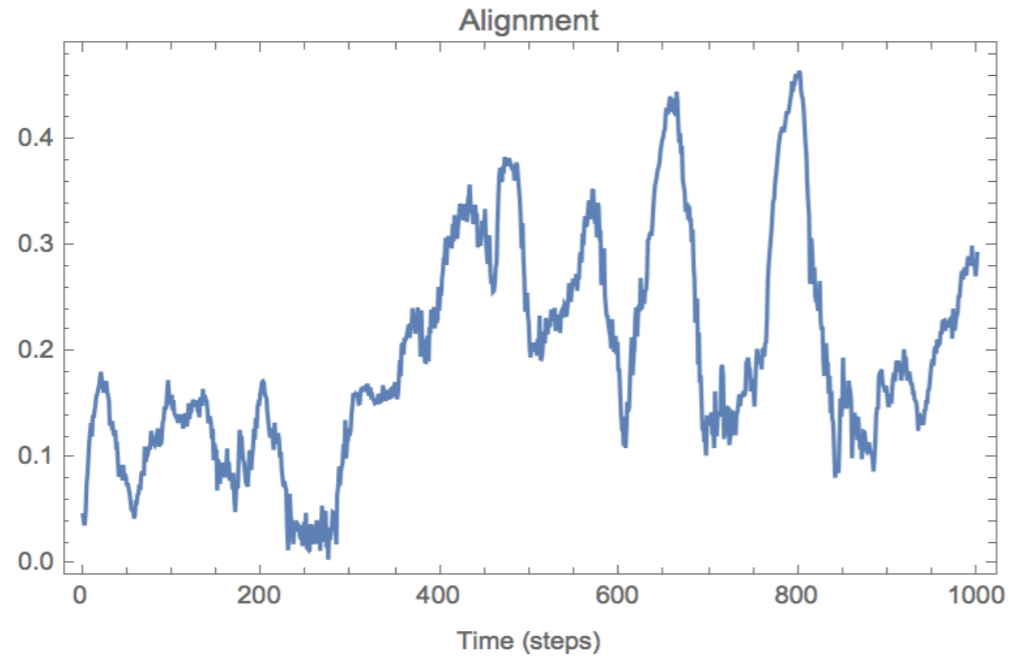
Square



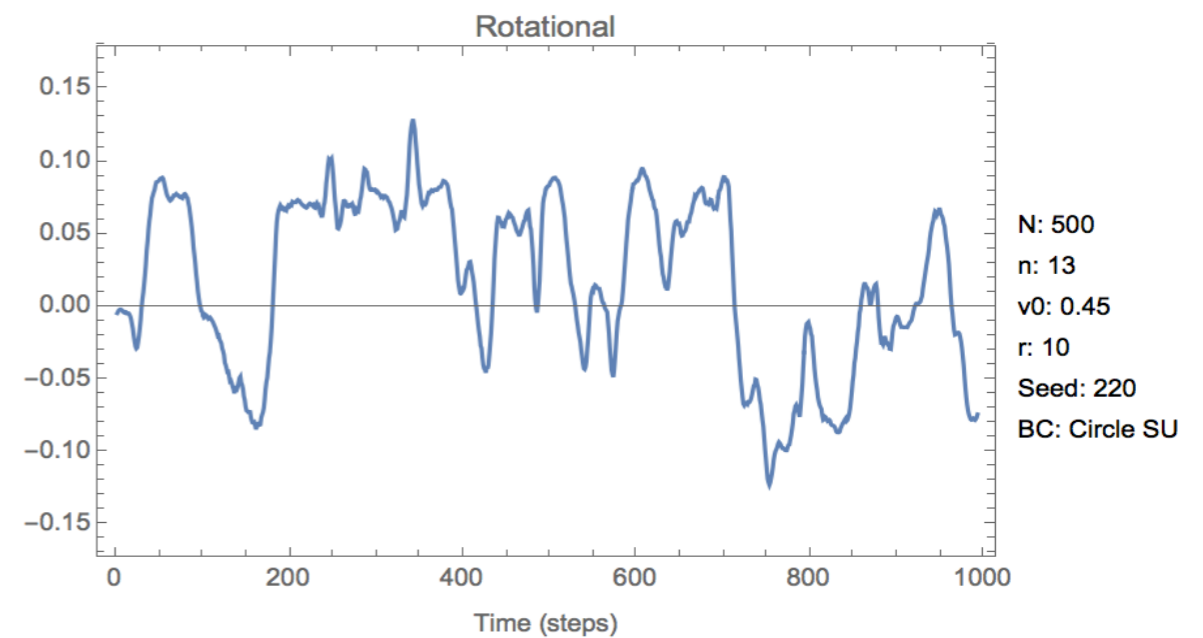
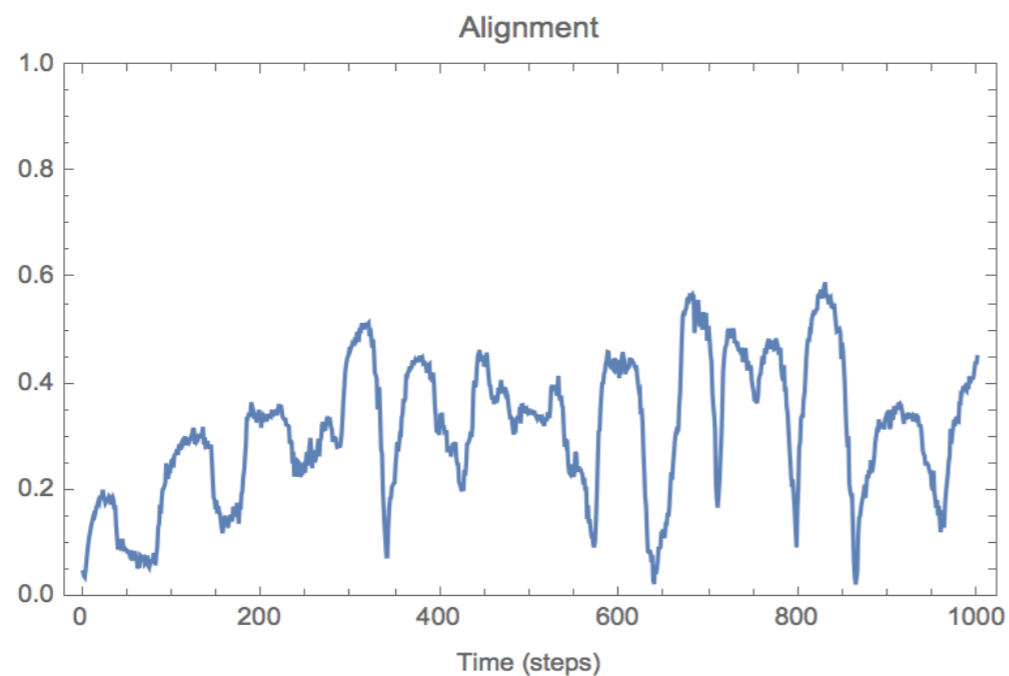
Circle

Comparing Boundary Conditions

**Square
U-Turn**

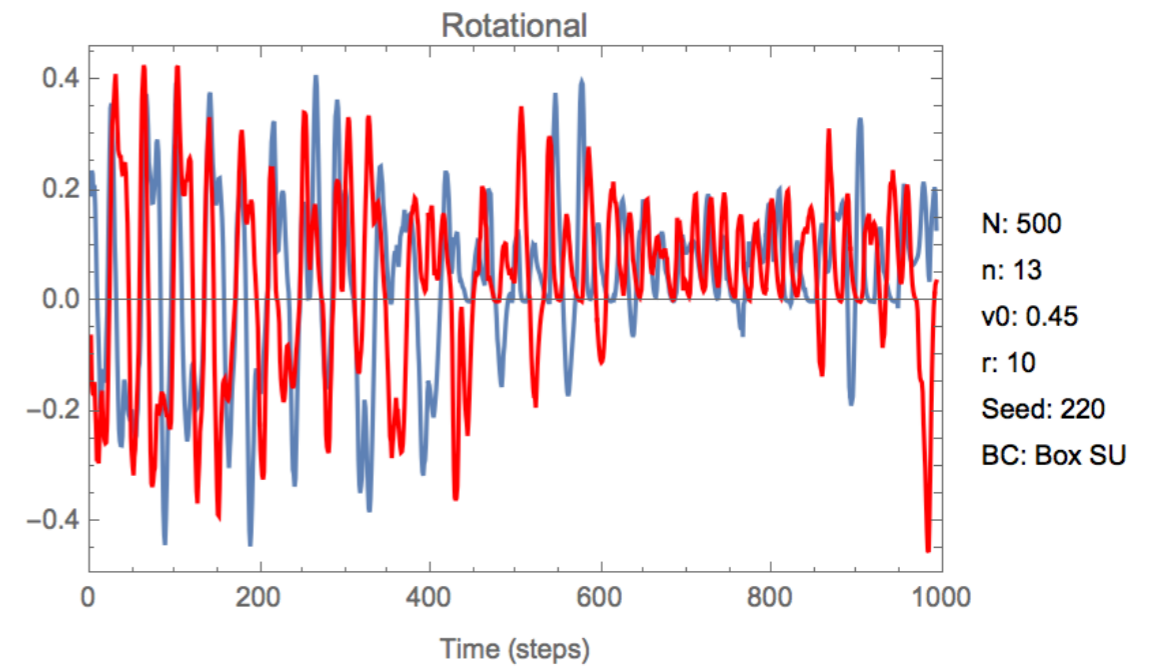
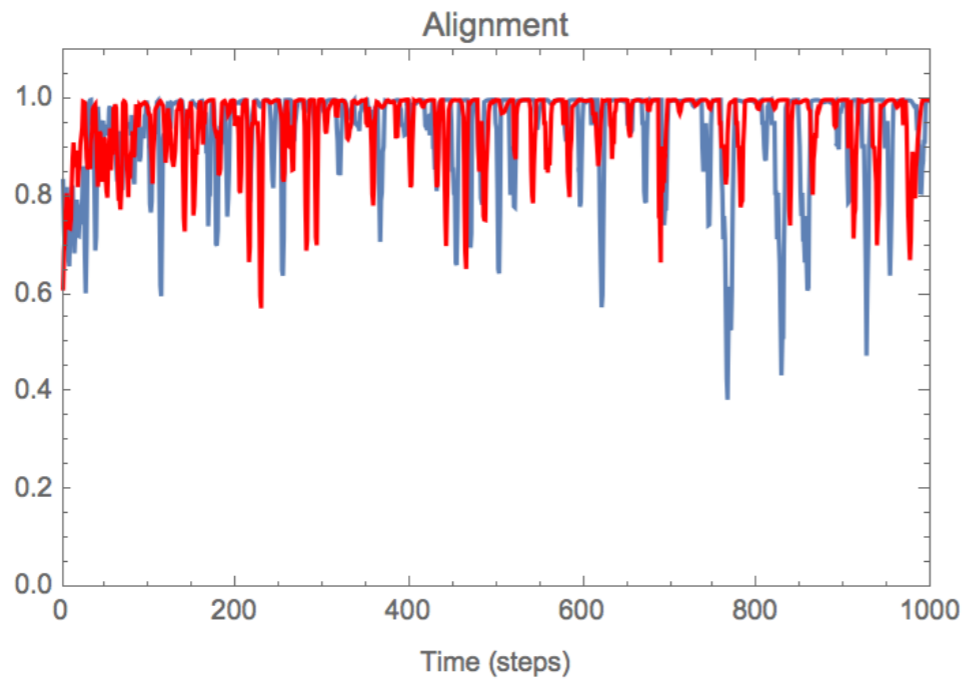


**Circle
U-Turn**

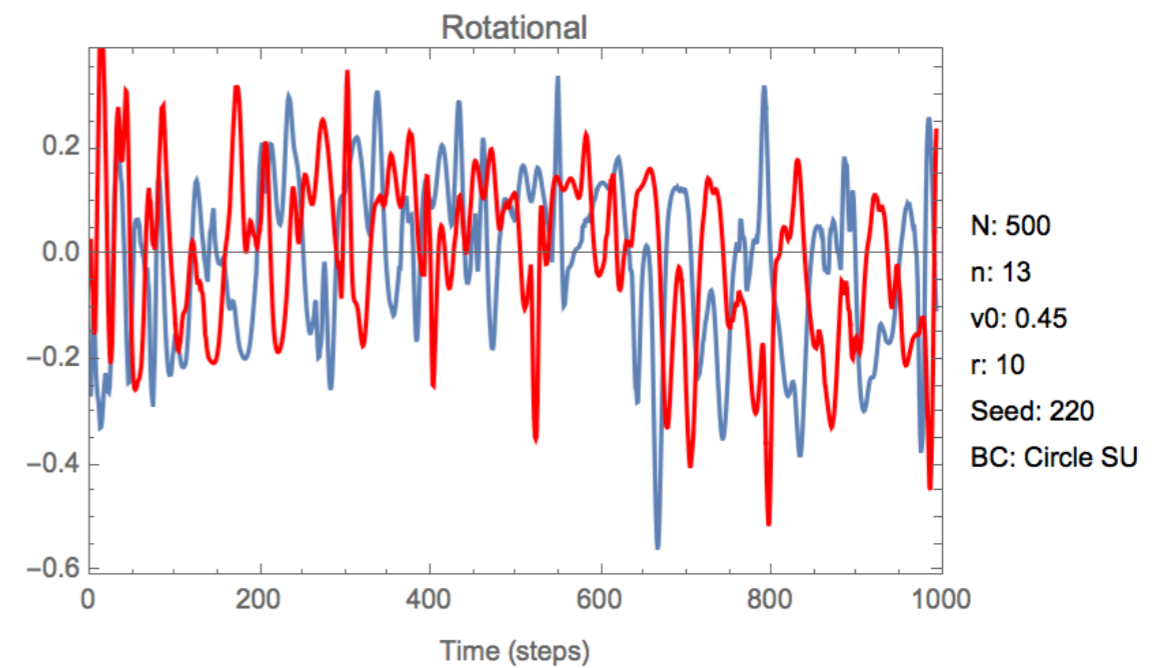
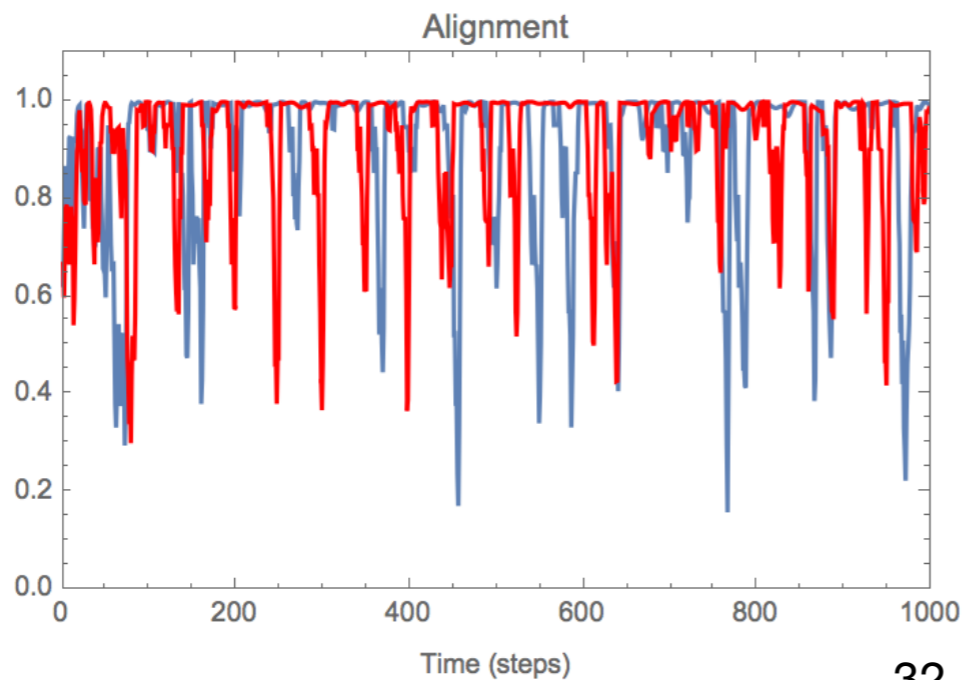


Comparing Boundary Conditions

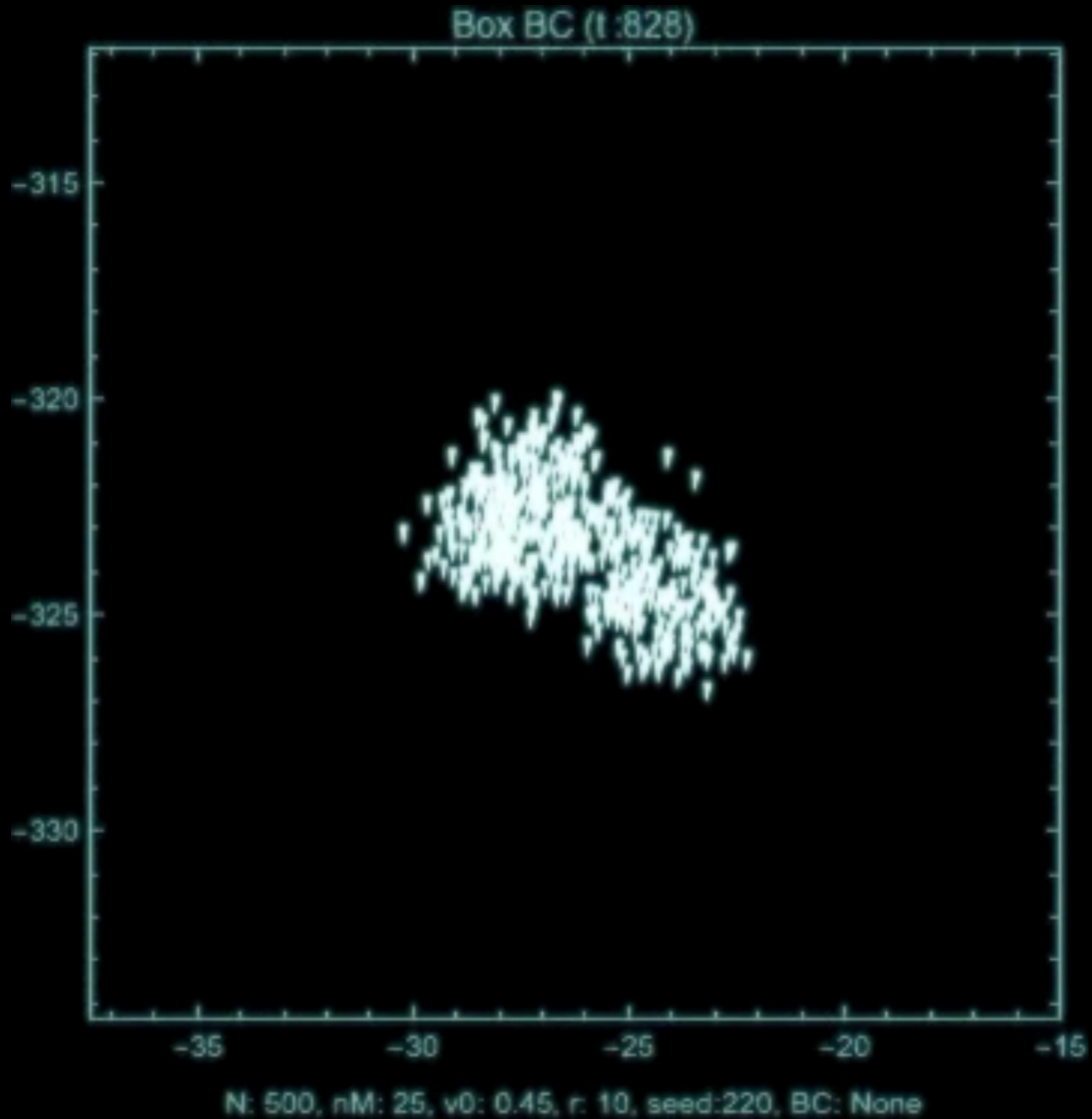
**Square
U-Turn**



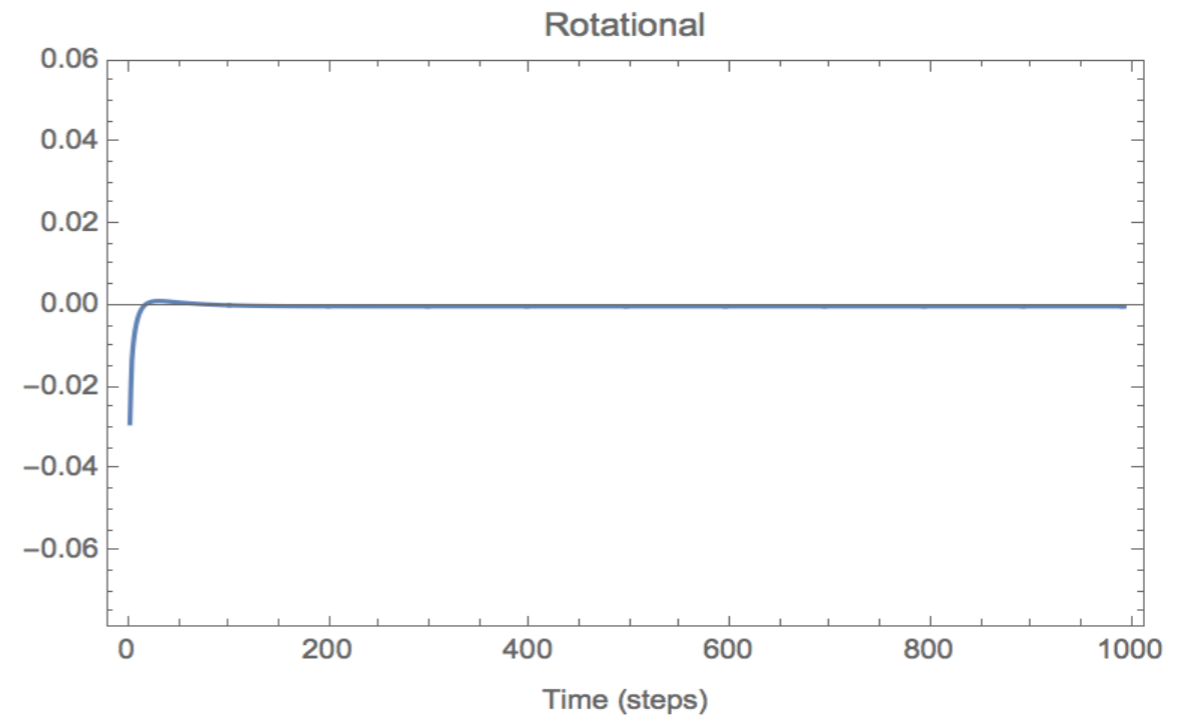
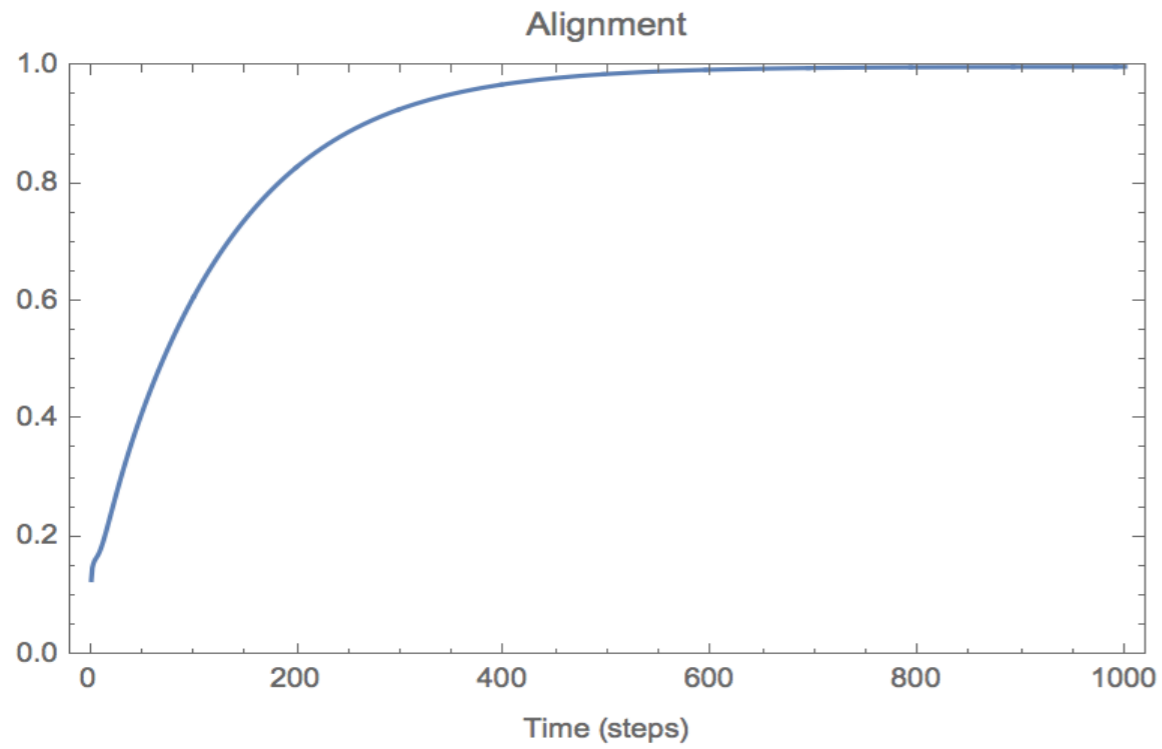
**Circle
U-Turn**



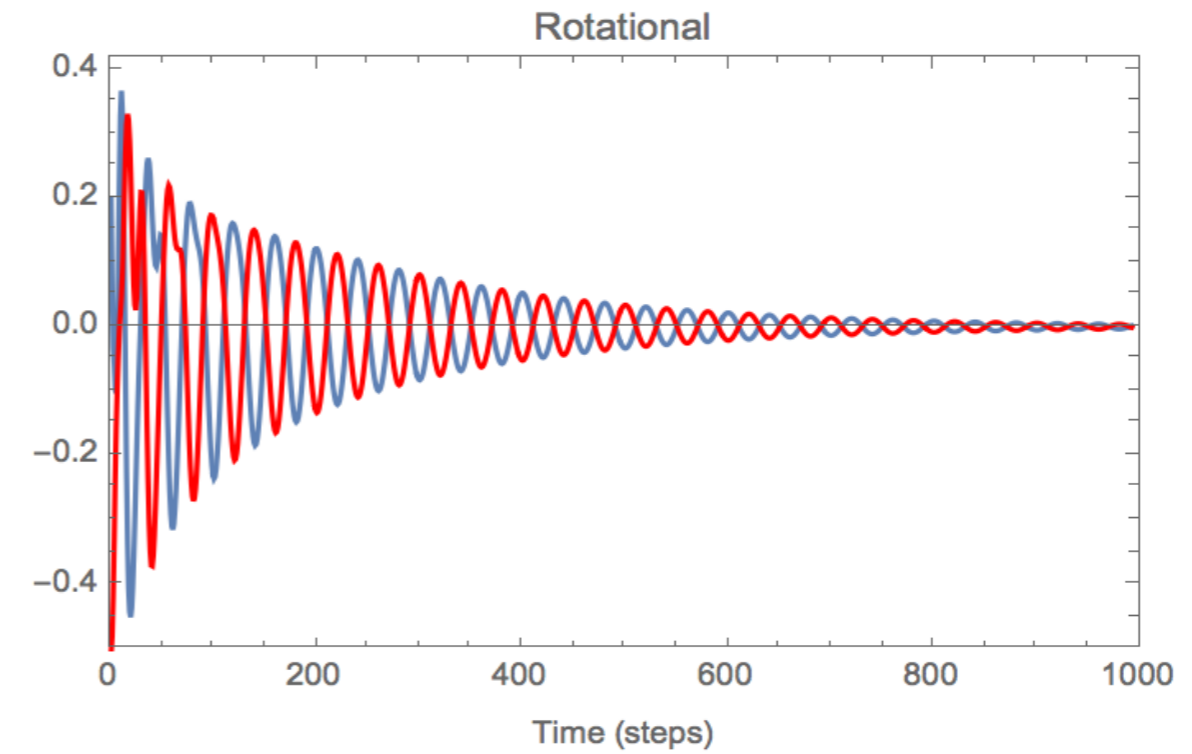
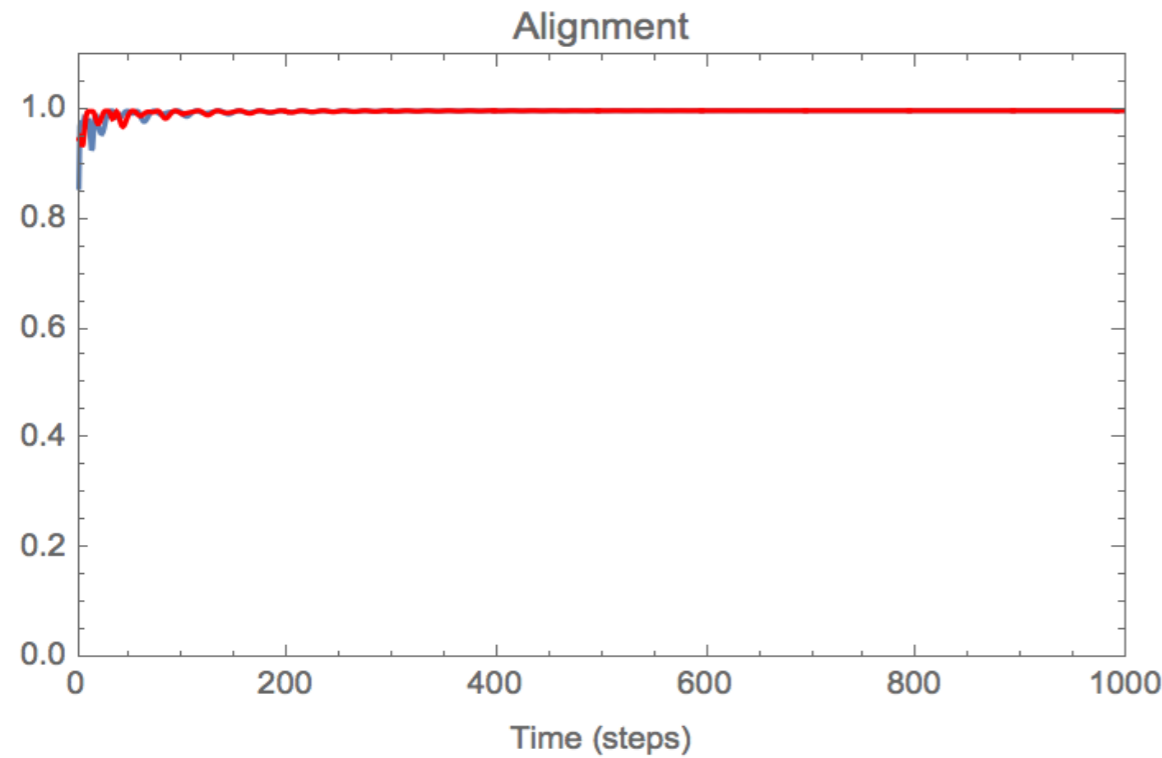
No Boundaries



No Boundaries



N: 500
n: 25
v0: 0.45
r: 10
Seed: 220
BC: None



N: 500
n: 25
v0: 0.45
r: 10
Seed: 220
BC: None

Periodic Boundaries

