# Quantifying the Effects of Stellar Flybys on Planetary Systems

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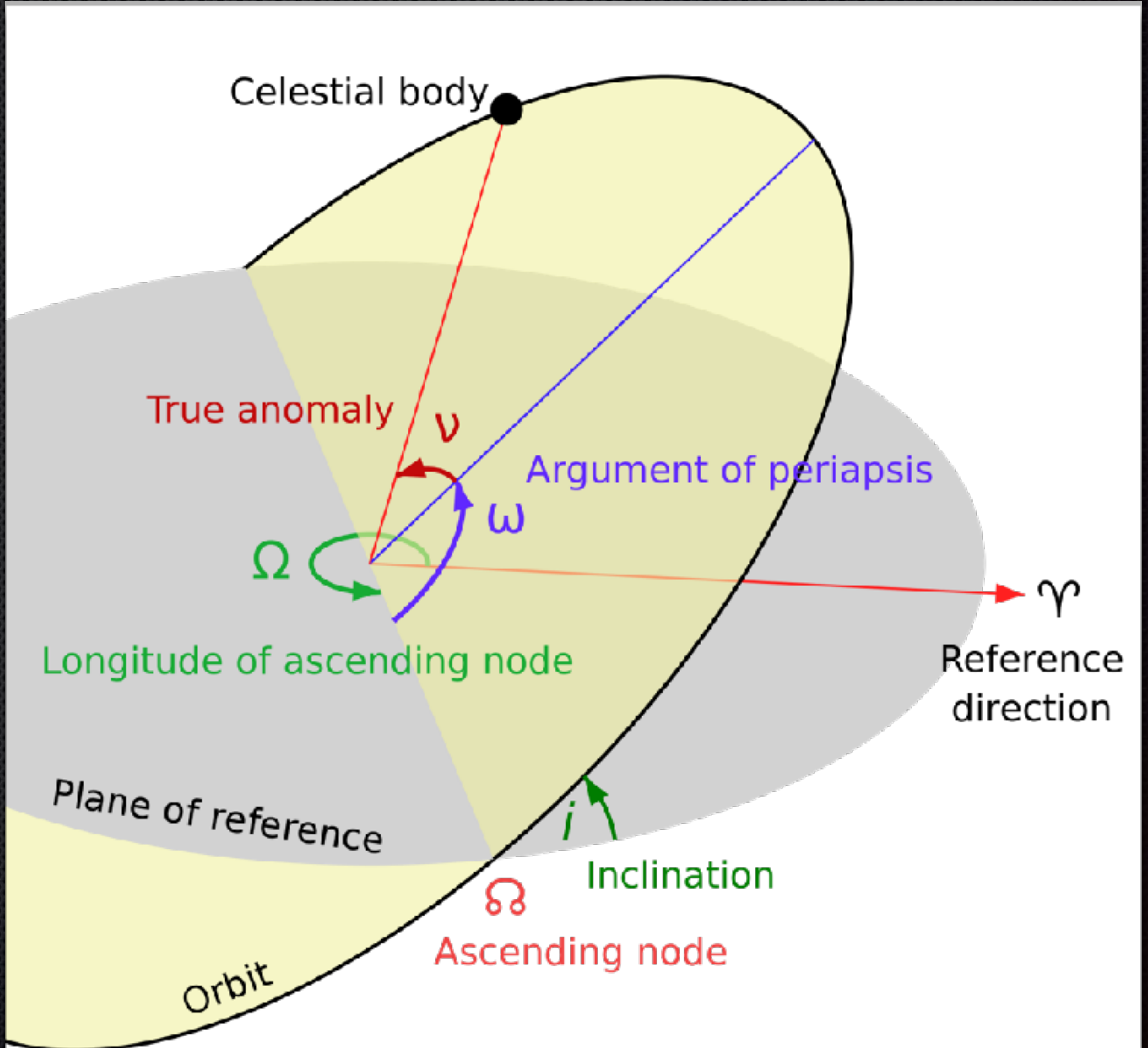
# Outline

## N-body simulations and Secular Dynamics

- Frequency Modified Fourier Transform (FMFT)
- Recent work by Spalding, Fischer, and Laughlin (2018).
- Numerical checks and optimizations for computing secular frequencies.
- Applications of FMFT to an external body passing the solar system on a hyperbolic orbit.

# Secular Frequencies

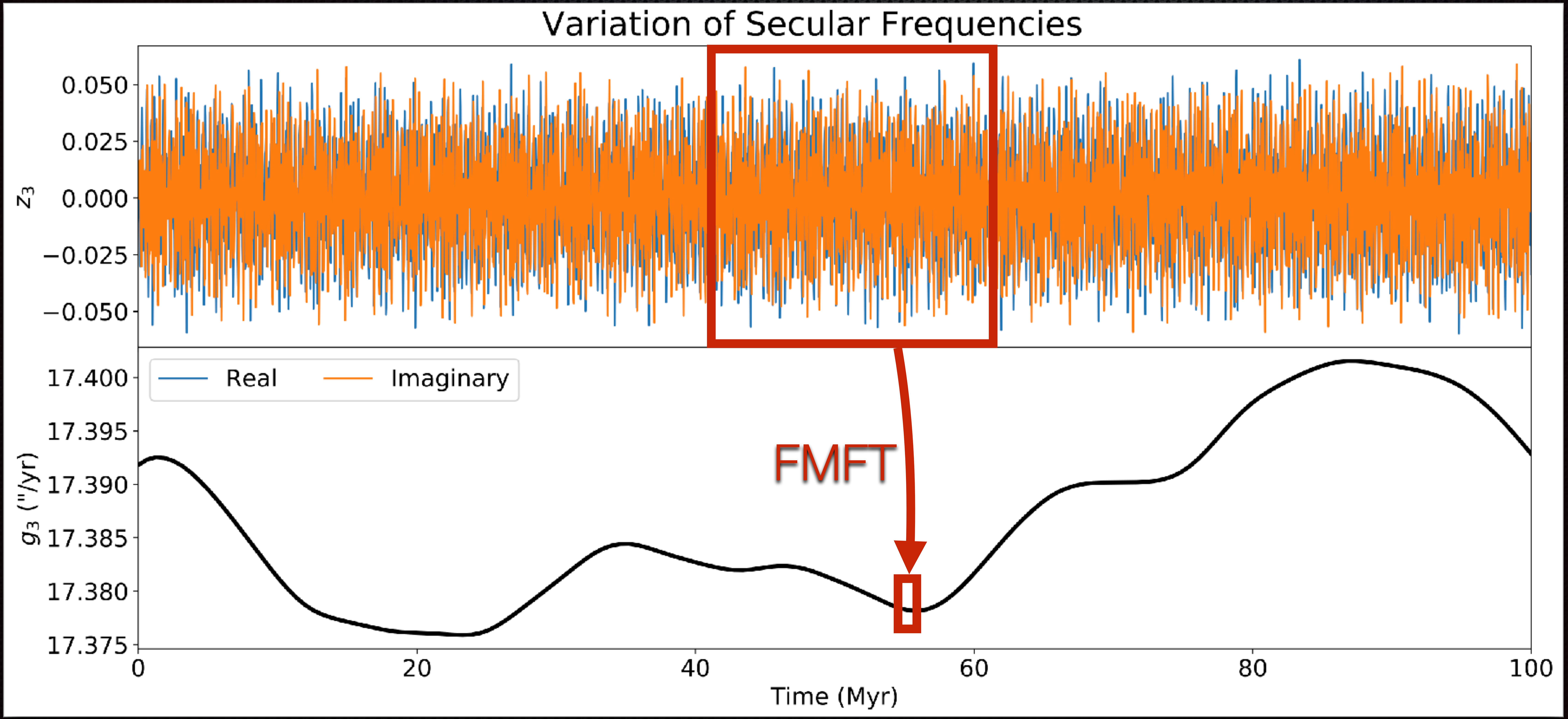
- Secular precessions of the periapses and the ascending nodes
- Obtained from time series data
  - $z_j = e_j \exp(\mathrm{i}\varpi_j)$
  - $\zeta_j = \sin(i_j/2)\, \exp(\mathrm{i}\Omega_j)$



Wikipedia/Orbital_elements

# Frequency Analysis

(Laskar 1988, 1990)
(Šidlichovský & Nesvorný 1997)



# Recent Work

## Spalding, Fischer, and Laughlin (2018)

- A quantitative test of solar mass loss.
- Linear solar mass loss of 5% over 4.5 Gyrs.
- Low inclinations and eccentricities.

$$g_i \propto n_i = \sqrt{\frac{GM_\star}{a_i^3}\frac{1}{M_\star(t)}} \qquad a_i \propto \frac{1}{M_\star} \qquad \Longrightarrow \quad g_i \propto M_\star$$

# Recent Work

## Spalding, Fischer, and Laughlin (2018)

- How persistent is the stability of the $g_2 - g_5$ mode?
- Does its frequency scale linearly with stellar mass?
- Tabulated every 450 yrs in Jacobi coordinates; FFT on 450 Myr segments.

The Astrophysical Journal Letters, 869:L19 (6pp), 2018 December 10
https://doi.org/10.3847/2041-8213/aaf219


CrossMark

**An Orbital Window into the Ancient Sun's Mass**

Christopher Spalding[1], Woodward W. Fischer[2], and Gregory Laughlin[1]
[1] Department of Astronomy, Yale University, New Haven, CT 06511, USA
[2] Division of Geological and Planetary Sciences, California Institute of Technology, Pasadena, CA 91125, USA
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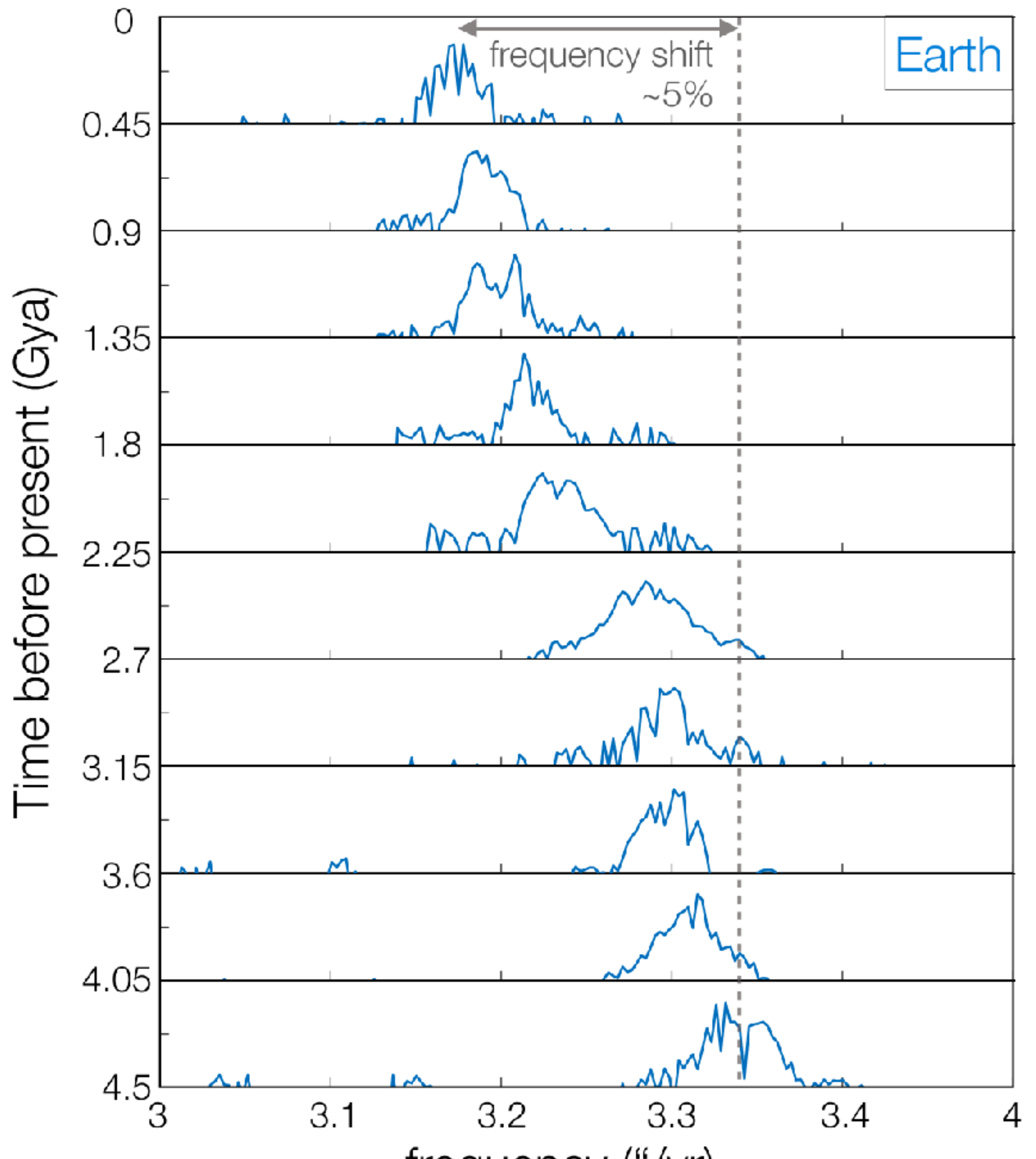
# Recent Work

Spalding, Fischer,
and Laughlin (2018)

- Frequencies do scale with mass.
- Matched fundamental secular modes of La2010 to within ~1%.
- FFT instead of FMFT

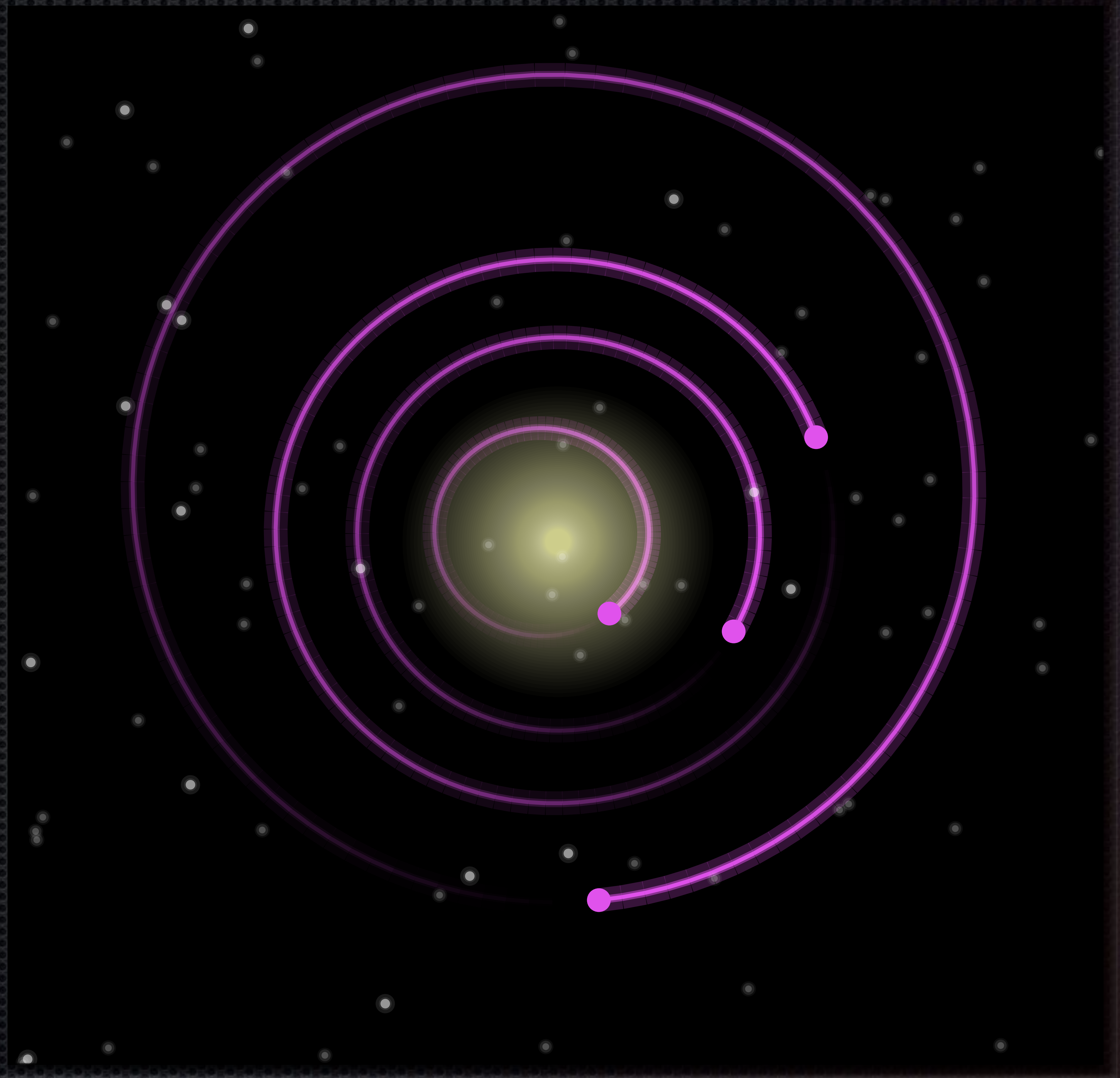


Spalding et. al. 2018

# Secular N-body

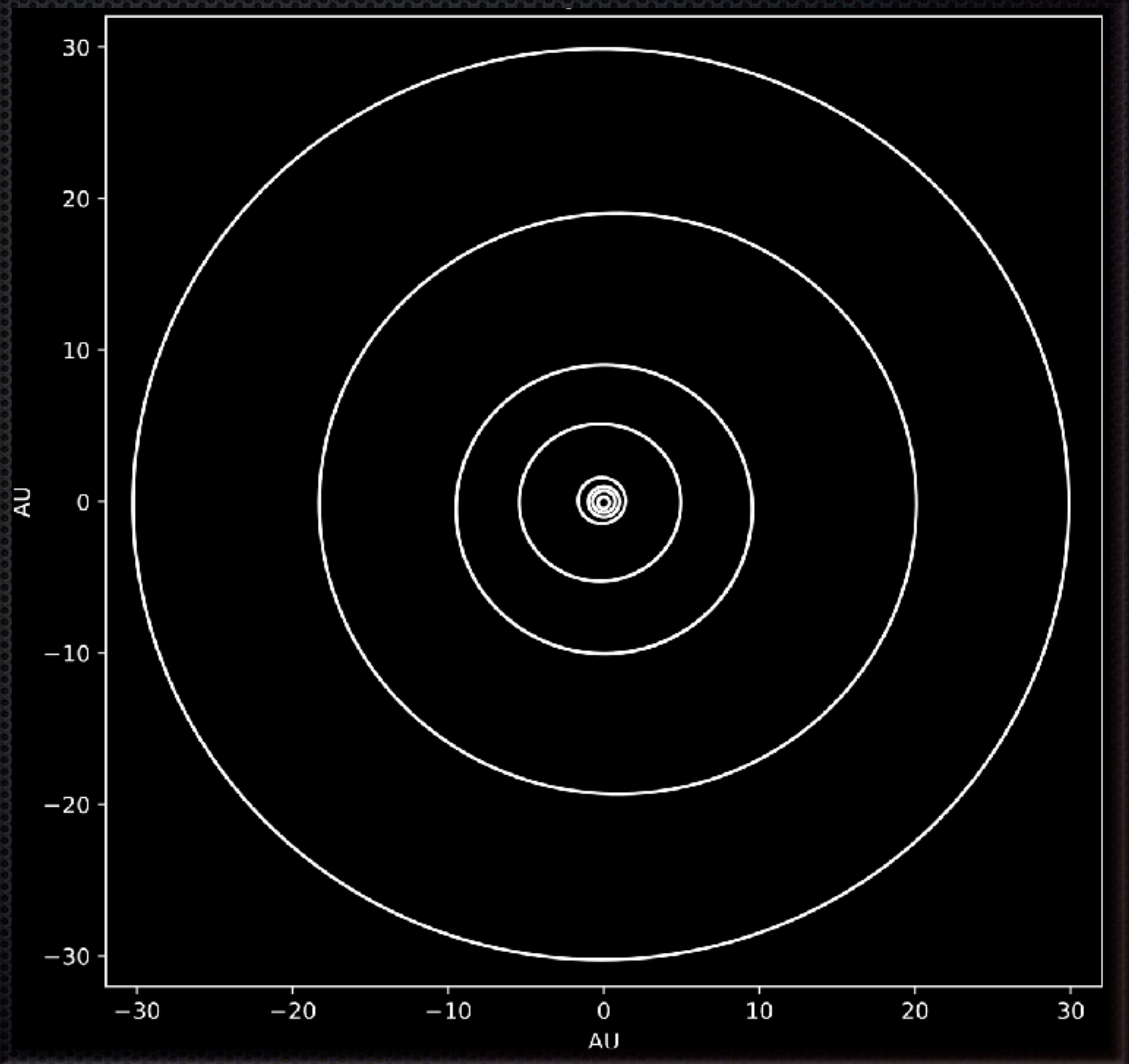
## REBOUND and La2010

- Understand the systematics
- Grasp the orders of magnitude



# Rebound

- Solar System (8 planets)
- NASA Horizons Database
- WHFast Symplectic Integrator
- GR scalar potential (Reboundx)



# Secular Frequencies

- Solar System fundamental frequencies
- Obtained with modified Fourier transform (MFT)
- 20 Myrs in ~25 minutes
- Inner planets within 0.1%
  Outer planets within 0.001%

| “/yr | RB2019 | La2010 |
|---|---|---|
| $g_1$ | 5.599 | 5.59 |
| $g_2$ | 7.420 | 7.453 |
| $g_3$ | 17.3582 | 17.368 |
| $g_4$ | 17.9163 | 17.916 |
| $g_5$ | 4.257523 | 4.257482 |
| $g_6$ | 28.24617 | 28.2449 |
| $g_7$ | 3.088024 | 3.087946 |
| $g_8$ | 0.673289 | 0.673019 |

(Laskar et al. 2011)

# Secular Frequencies

- Explored the effects of:
  - Window Length
  - Timestep
  - Sampling Frequency (Nyquist)
  - Coordinate System
  - Windowing Function

| "/yr | RB2019 | La2010 |
|---|---|---|
| $g_1$ | 5.599 | 5.59 |
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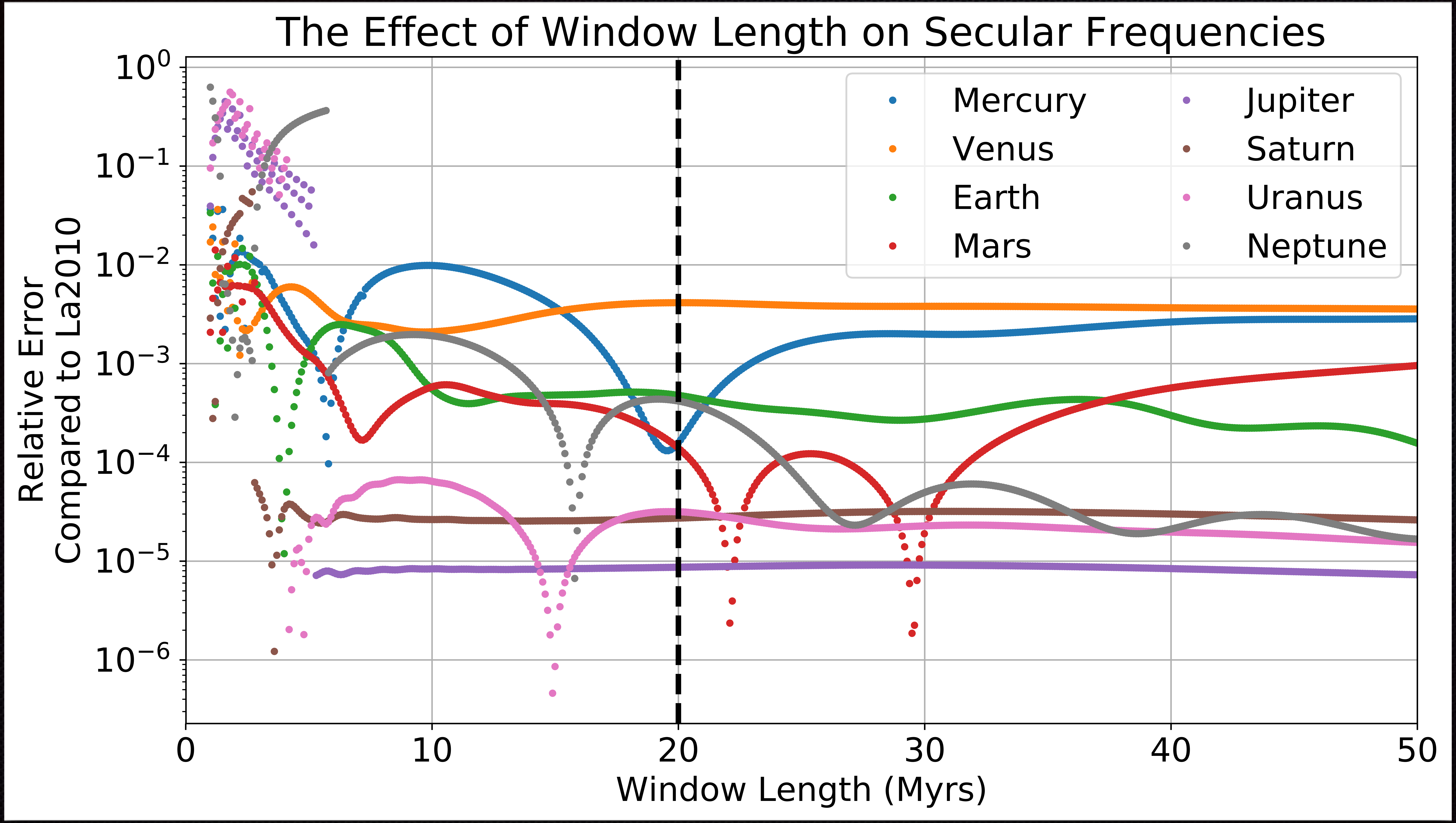
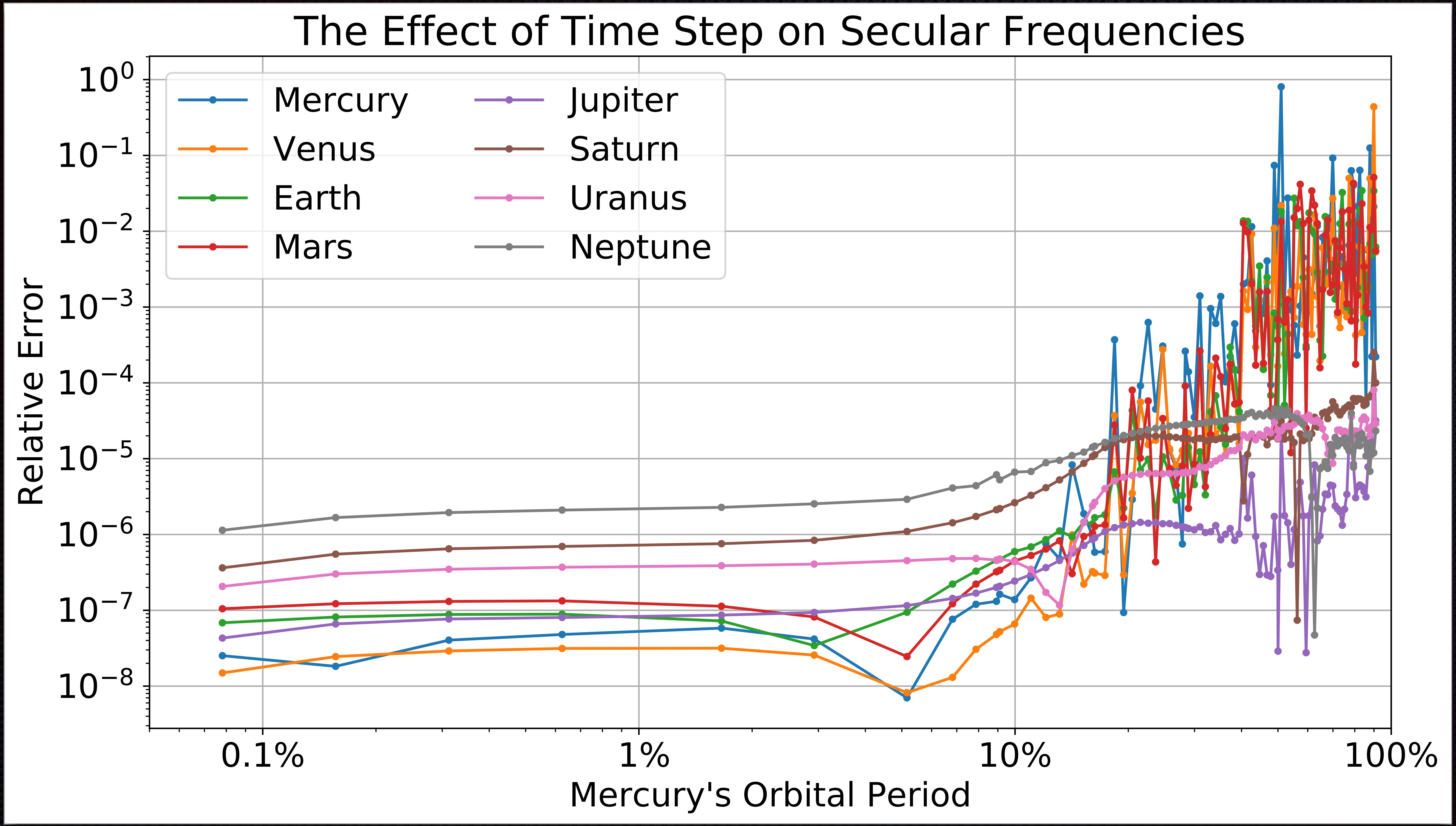



# Window Length

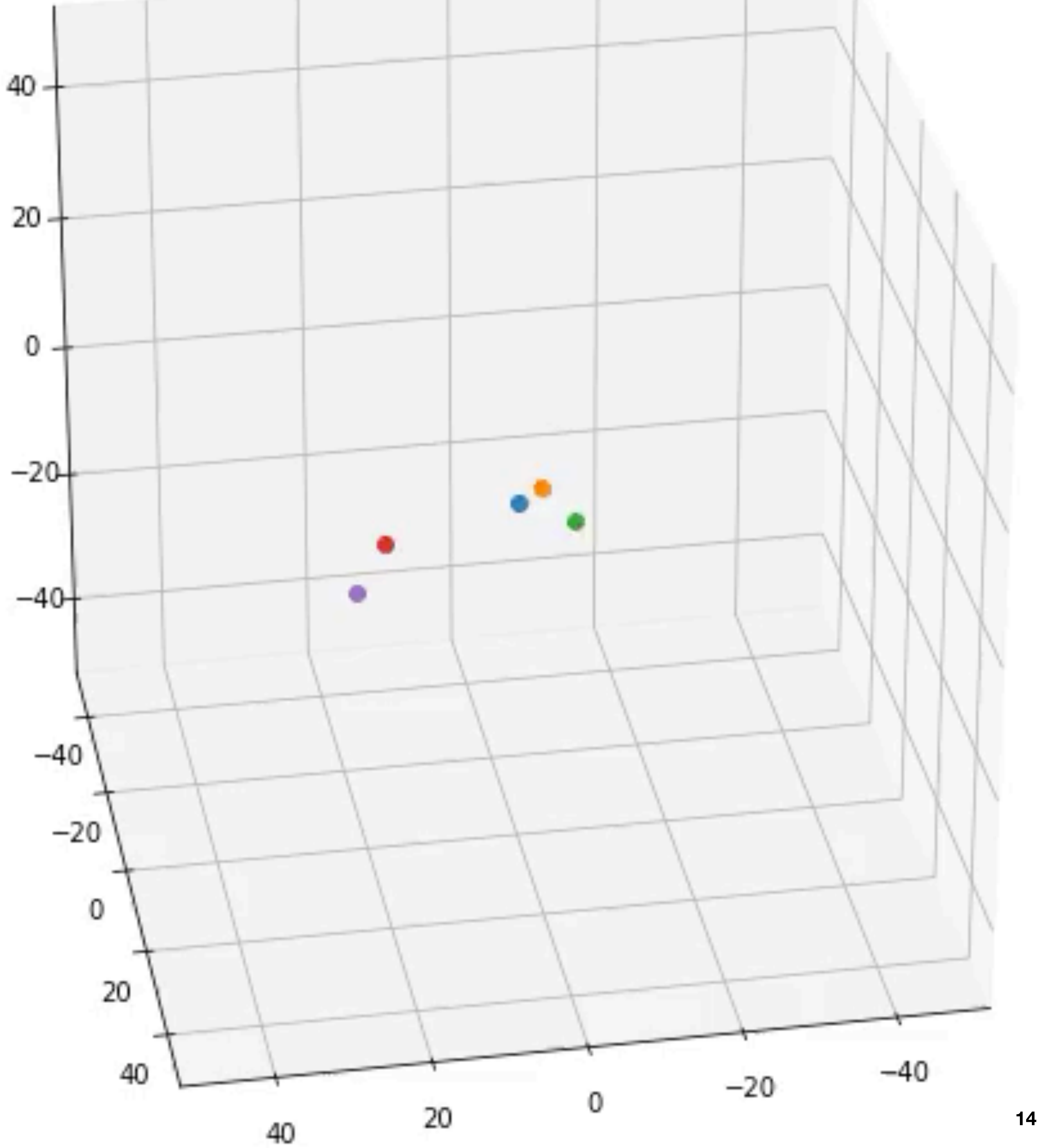



# Time step

## N-body Accuracy

# Quantifying Stellar Flybys

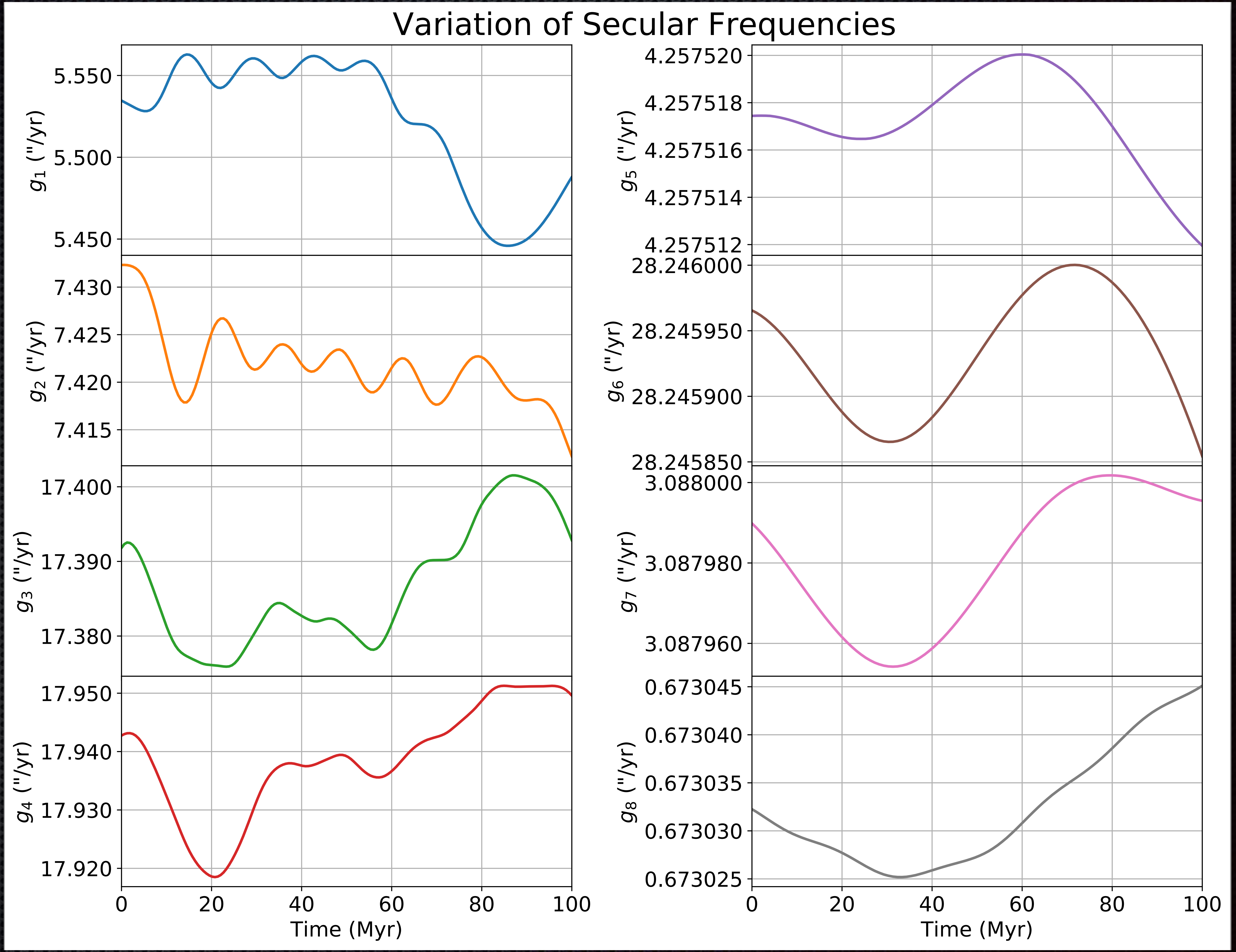
- Quantitative laboratory for the long-term stability of planetary systems.
- Exploration of changes to initial conditions.



# Quantifying Stellar Flybys

## No Perturbation

- Variations of secular frequencies over 100 Myrs
- Analyzed with FMFT

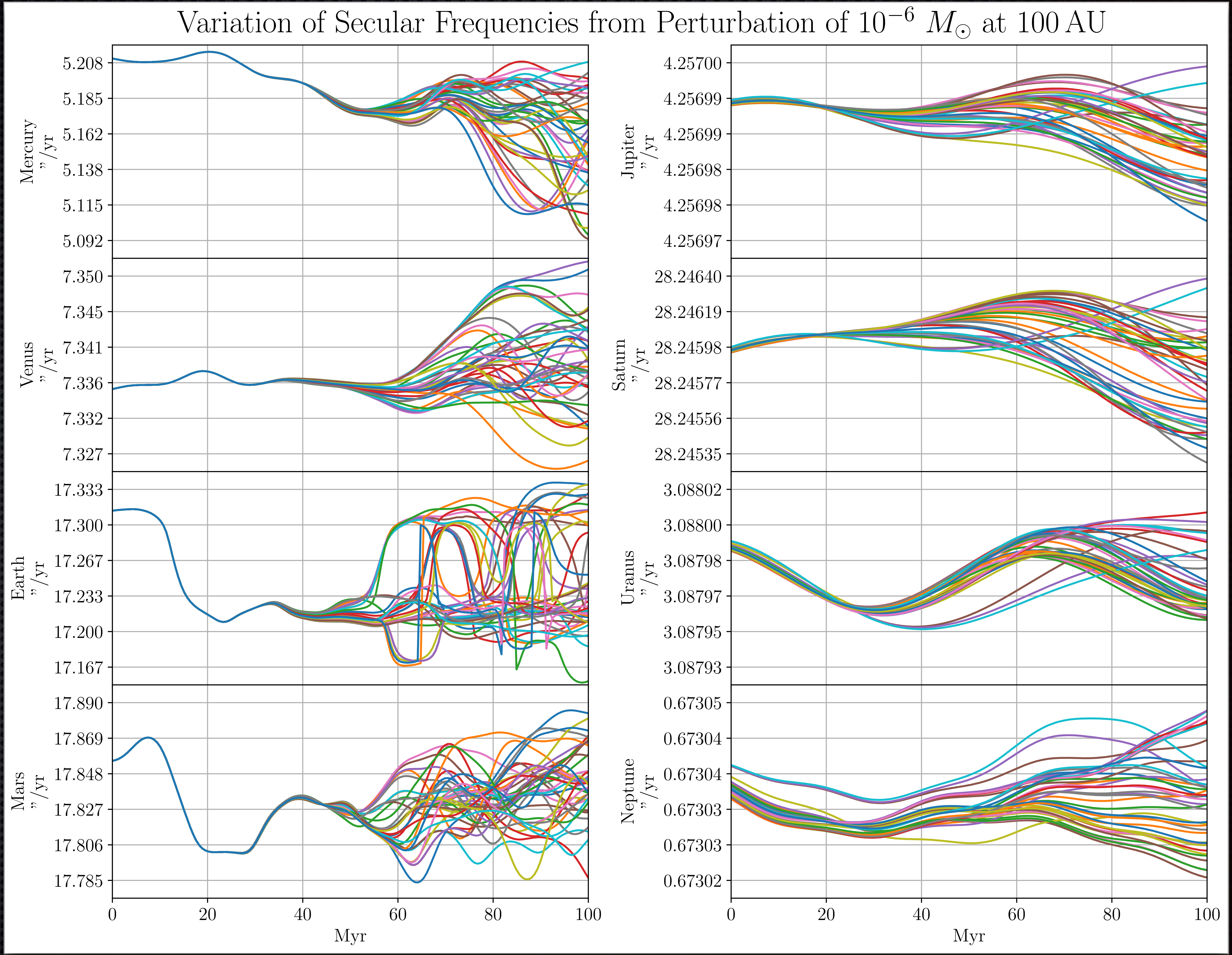



Brown and Rein, in prep.

# Quantifying Stellar Flybys

## Perturbation by Earth-mass at 100 AU

- Variations of secular frequencies over 100 Myrs
- Analyzed with FMFT




Brown and Rein, in prep.

# Conclusion

- The recent work by Spalding et al. aimed to identify the scaling between Solar mass and $g_2 - g_5$, together with the mode's stability.
- The secular frequencies of the solar system can be computed from N-body integration in ~25 minutes (with GR corrections).
- Using FMFT, very subtle changes made to the solar system can be detected.

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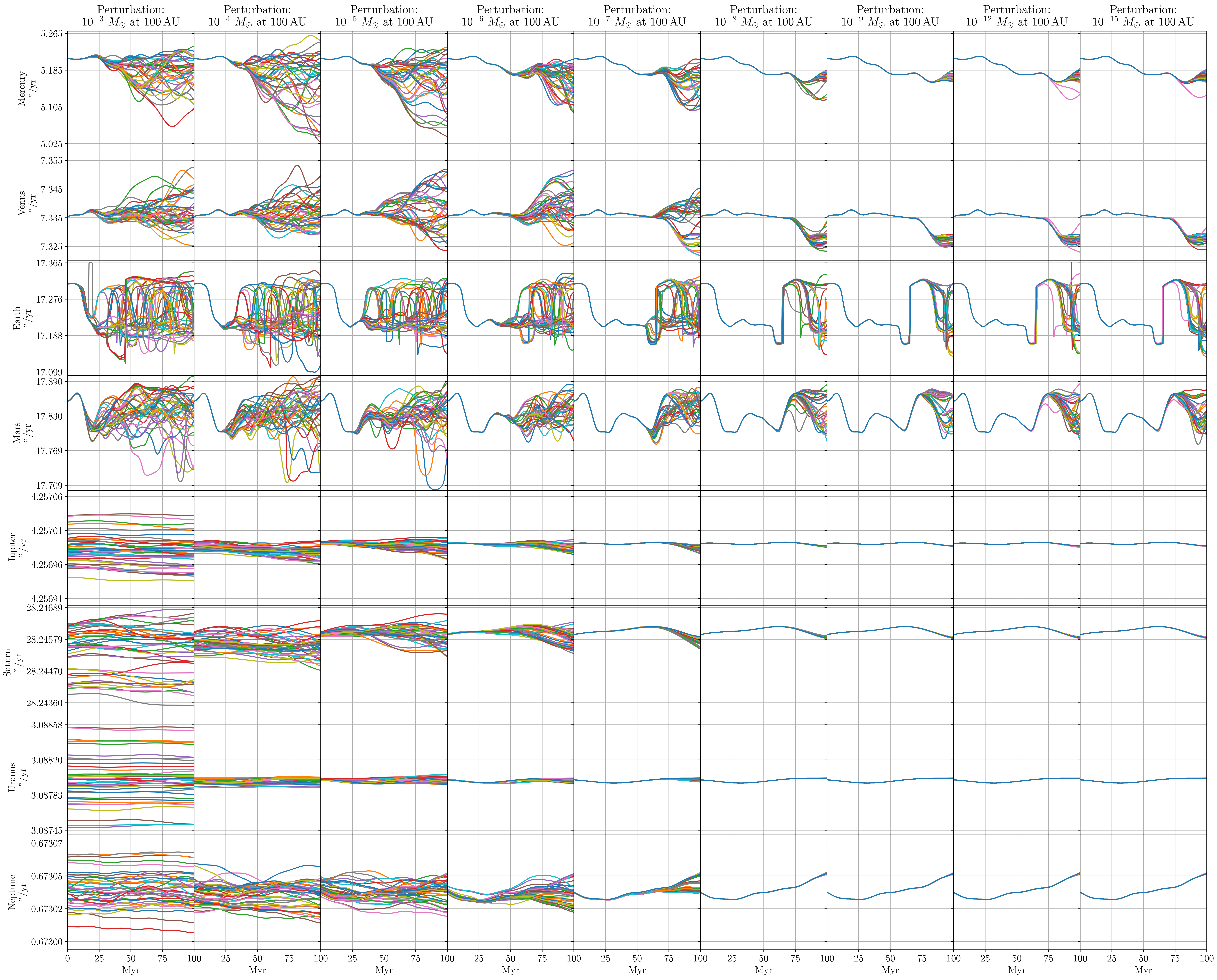
- Decreasingly massive perturbations
- FMFT




Brown and Rein, in prep.

(Laskar 1988, 1990)

# Modified Fourier Transform

## A simple example

$$y(t) = \exp(6.28\mathbf{i}\, t),\ t \in [0,10],\ N = 1000$$

6.276902122

$$\phi(\nu) = \langle y, \exp(\mathbf{i}\nu t)\rangle$$

6.280000000